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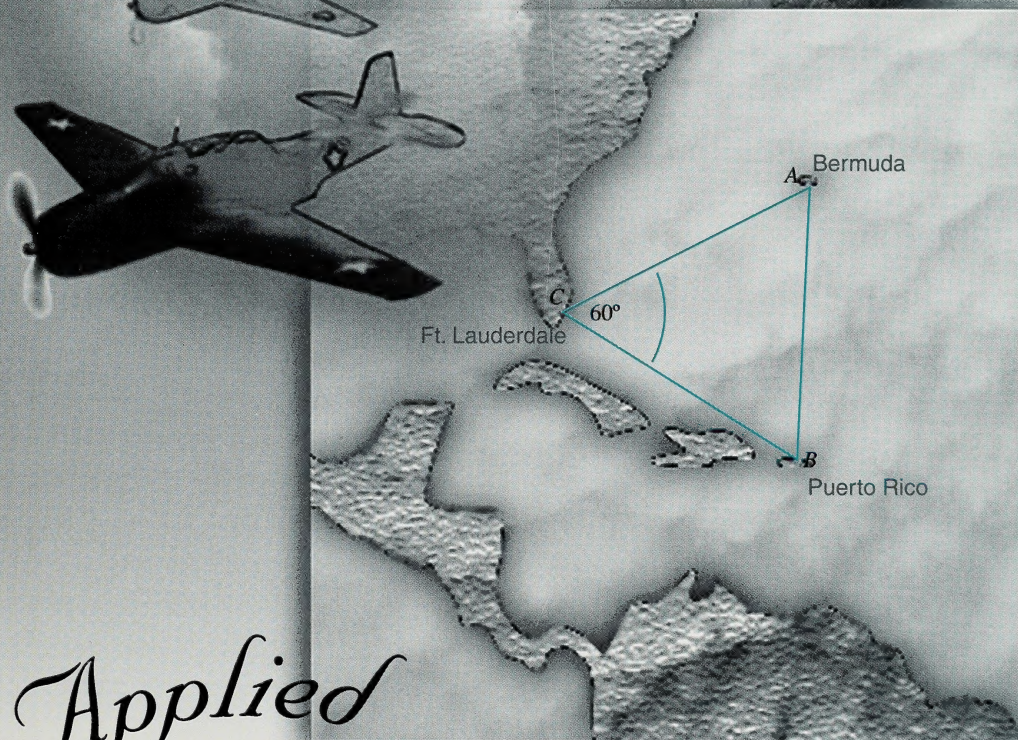
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Module 7

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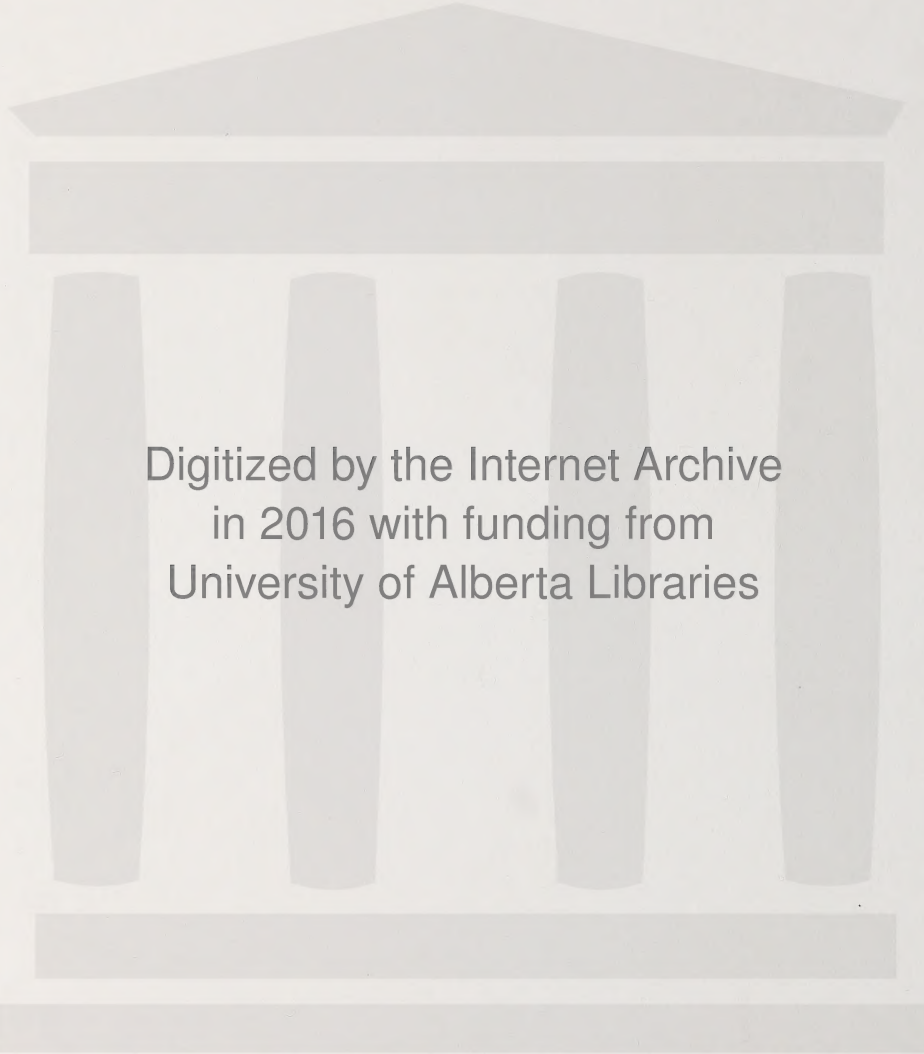
Mathematics 10

TRIGONOMETRY



Learning
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Module 7



Applied

Mathematics 10

TRIGONOMETRY

Applied Mathematics 10
Student Module Booklet
Module 7
Trigonometry
Learning Technologies Branch
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	



The Learning Technologies Branch has an Internet site that you may find useful.
The address is as follows:

<http://www.learning.gov.ab.ca/ltb>

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

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Welcome

Applied Mathematics 10

Module 1
Measurement

Module 2
Number Patterns in Tables

Module 3
Relations and Functions

Module 4
Sampling

Module 5
Line Segments

Module 6
Linear Functions

Module 7
Trigonometry

*Welcome
to Module 7.
We hope you'll
enjoy your study
of Trigonometry.*



Applied Mathematics 10 contains seven modules and a final test. Work through the modules in the order given, since several concepts build on each other as you progress through the course.

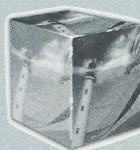
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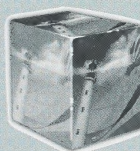
*Submit the
Module Project.*



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*Submit the
Module Assignment.*



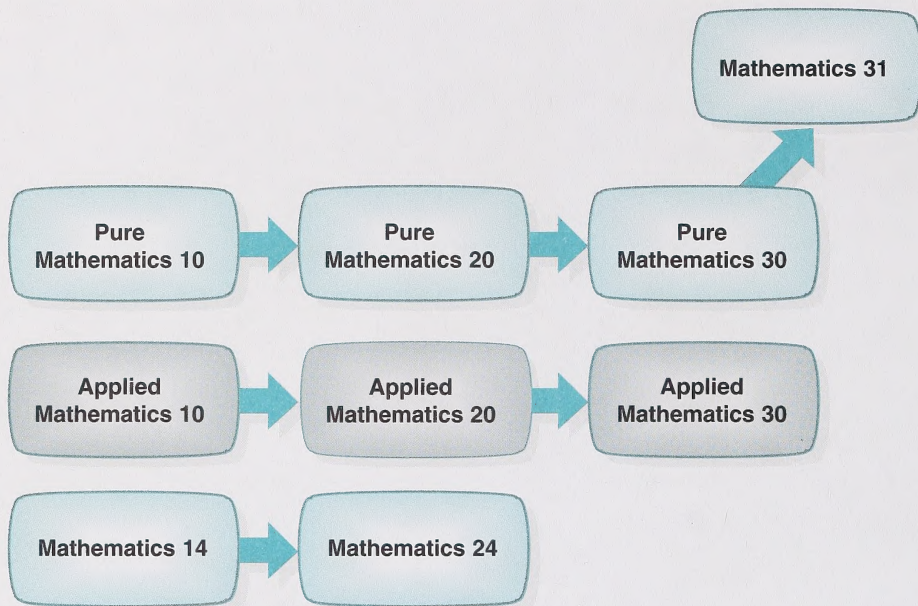
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Introduction to Applied Mathematics 10

Applied Mathematics 10 is the first course in the Applied Mathematics 10–20–30 program of studies. Another program of studies is Pure Mathematics 10–20–30; students who complete Pure Mathematics 30 often choose to take Mathematics 31. A third program of studies is Mathematics 14–24.



Each mathematics program is designed for students with different mathematical strengths and interests.

- Pure Mathematics 10–20–30 is intended for students who are strong in algebra and mathematical theory.
- Applied Mathematics 10–20–30 is better suited to students who prefer to solve problems using numerical reasoning or geometry.
- Mathematics 14–24 is a general mathematics program for high school students who have experienced difficulties in previous mathematics courses.

Each sequence of courses is designed for students with different post-secondary and career plans.

You may find it helpful to read mathematics updates on Alberta Learning's website:

<http://www.learning.gov.ab.ca>

You may also wish to visit the website for the Learning Technologies Branch:

<http://www.learning.gov.ab.ca/ltb>

Before enrolling in Applied Mathematics 10, it is recommended that you talk with a school counsellor about your career plans.



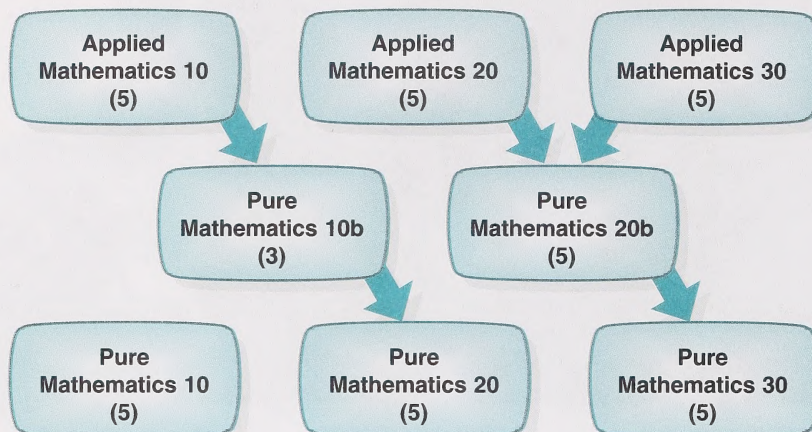
TRANSFERRING FROM THE APPLIED MATHEMATICS PROGRAM

You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are specific to either the applied or pure mathematics courses.

The following table shows some of the common topics and some of the independent topics in the mathematics program.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • surveys • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectation • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

If you want to transfer from the Applied Mathematics 10–20–30 sequence to the Pure Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics. If you decide to transfer to Pure Mathematics 20 after successfully completing Applied Mathematics 10, you may take the 3-credit course called Pure Mathematics 10b. If you decide to transfer to Pure Mathematics 30 after successfully completing Applied Mathematics 20 or Applied Mathematics 30, you may take the 5-credit course called Pure Mathematics 20b. The two bridging courses are shown in the following diagram.



STRATEGIES FOR COMPLETING APPLIED MATHEMATICS 10

For each module in Applied Mathematics 10, there is a Student Module Booklet, a Project Booklet, and an Assignment Booklet. The document you are presently reading is called a Student Module Booklet.

Each Student Module Booklet will show you, step by step, what to do and how to do it. There are readings, questions for you to answer in your mathematics binder, and applications that will give you hands-on experience.

It is important to work systematically and carefully through the Student Module Booklets. This work will prepare you for the projects, assignments, and final test.

Following are some suggestions for organizing your mathematics binder:

- Keep a section of your binder to record your responses to the questions in the Student Module Booklet. Also, store your marked assignments here.
- Keep a section of your binder for work in progress on your projects. Keep your research notes, plans, rough drafts, and so on.
- Keep a section of your binder to record new skills and concepts as well as important results and formulas. Get in the habit of describing new skills and concepts in your own words, and recording useful ways to help you remember what a concept means. Make charts and diagrams to help you connect mathematical ideas.
- Keep a section of your binder to record mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you grasped a difficult concept (an “aha!” experience) or when you used a calculator or spreadsheet in a new way.

Mathematical Processes

Throughout this course, you will be expected to perform the following mathematical processes:

- Connect mathematical ideas to everyday experiences and to concepts in other disciplines.
- Develop and use problem-solving strategies.
- Reason and justify your answers.
- Communicate mathematical ideas.
- Select and use appropriate technologies to solve problems.
- Develop and use estimation and mental-math strategies.
- Use visualization to assist in processing information, making connections, and solving problems.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Applied Mathematics 10 to be your study partner. You will find that having a friend with whom you can discuss mathematical ideas will make your studying more enjoyable.



Resources You Will Need

In addition to the distance learning materials for Applied Mathematics 10, you will need the following resources:

- the *Addison-Wesley Applied Mathematics 10 Source Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (1999)
- a binder, lined loose-leaf paper, graph paper, dividers, a pencil, and an eraser
- metric and imperial measuring devices, such as a ruler, a tape measure, a yardstick, a vernier caliper, and a micrometer
- a mathematical instrument set (compass, protractor, and triangles)
- a computer installed with a spreadsheet program

Note: Two popular spreadsheet programs are *ClarisWorks™* and *Microsoft® Excel*. The examples in this course show *Microsoft® Excel*.

- a graphing calculator

Note: Where it is applicable, the examples in this course and the textbook show the TI-83 calculator; however, all of the graphing calculators in the following chart are approved for use on tests.

Brands	Texas Instruments	Sharp	Casio
Models	TI-82 TI-83 TI-83 Plus TI-86 TI-89 TI-92 TI-92 Plus	EL-9600c EL-9600* EL-9300* EL-9200*	CFX-9850Ga-Plus CFX-9850G* CFX-9800G* FX-9700 series*

*no longer commercially available, but may be available on loan from your school division

If you intend to use the TI-83 or TI-83 Plus graphing calculator, it is recommended that you view the video *The TI-83 Graphing Calculator Video Tutor* to discover some of the calculator's features.

Many of the resources you will need for this course may be purchased from the Learning Resources Distributing Centre (LRDC). Following is the LRDC website:

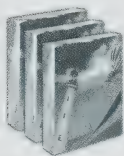
<http://www.lrdc.edc.gov.ab.ca>

You may wish to discuss the availability of resources with your teacher, as your school division may have a loan policy.

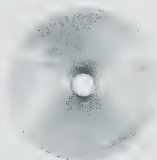
Visual Cues

You will find many visual cues in this course. Colour is used to highlight terms that are defined in the Glossary of the Appendix of each Student Module Booklet. You will also find several icons in the margins. Read the explanations given to discover what the various icons prompt you to do.

- Refer to the textbook.



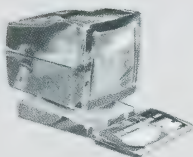
- Use the companion CD for Applied Mathematics 10.



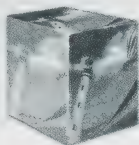
- Use mathematical instruments, measuring devices, and other materials.



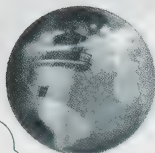
- Work with a computer.



- Complete the module project or assignment.



- Explore the Internet.



Remember: Any Internet website address given in this module is subject to change.

MODULE OVERVIEW



NOTMAN PHOTOGRAPHIC ARCHIVES, MCCORD MUSEUM OF CANADIAN HISTORY, MO

Building a railroad across Canada was a major engineering feat. On July 4, 1886, the first trans-Canada train completed its journey from Montreal, Quebec, to Port Moody, British Columbia. The civil engineers of the time accomplished “the impossible.”

Just as in the past, engineers today use a transit (also called a transit theodolite or theodolite) to determine exact land boundaries. They then use a branch of mathematics called trigonometry to calculate the lengths of the sides and the angles of a property and the location of any structure(s) on the land.

Roads, your home, buildings, and other structures have all been built using the basic principles of trigonometry.

In this module, you will investigate the mathematics of trigonometry and discover how trigonometry is used to determine the sides and angles of triangles. You will use what you learn to solve problems involving trigonometric ratios. You will build a clinometer and a transit that you will use to survey a given area.

EVALUATION

Accompanying this Student Module Booklet is a Project Booklet and an Assignment Booklet. Your grading in this module will be based upon the module project and the module assignment you submit for evaluation.

The mark distribution is as follows:

Module Project	40 marks
Module Assignment	<u>60 marks</u>

TOTAL 100 marks

Remember that Activities 1 to 6 in this Student Module Booklet will prepare you for the module project and the module assignment. You should work through these activities carefully and compare your answers with the suggested answers provided in the Appendix.

The Follow-up Activities provide extra help and enrichment. You may choose to do some or all the questions in the Follow-up Activities. Again, you should compare your answers with the suggested answers provided in the Appendix.

MODULE PROJECT

LAND SURVEYING



Beginning the Project

Your project for Module 7: Trigonometry, is called Land Surveying. This project involves building a clinometer and a transit, and then using these instruments to do a land survey of an area of your choice. The land survey will consist of determining the exact boundaries of the land you choose. In this project, you will also determine the heights of any buildings, trees, and/or other structures on the property.

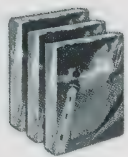
Turn to page 320 of the textbook and read “Land Surveying.”

Complete the questions posed. Store your answers in the project section of your mathematics binder. You may not be able to answer all or very many of the questions at this time. Return to your project work from time to time to answer these questions more fully.

Next, research on the Internet or at a library some of the things that you may need to know when doing a land survey. You may wish to begin your research by visiting Addison-Wesley’s Internet site described on page 321 of the textbook.

You may also wish to visit the Museum of Surveying to find more information:

<http://www.surveyhistory.org>



To do the module project, Land Surveying, you will need to build and use a **clinometer** and a **transit**. Turn to pages 328 and 329 in the textbook and read “Making and Using a Clinometer and a Transit.” Read the information carefully. **Note:** You will find printed 180° and 360° protractors at the end of this Student Module Booklet. Use these printed protractors and follow the instructions in the textbook to build your clinometer and transit. (Extra tips on making and using these instruments are given in the Extra Help.) You will need to glue the printed protractors to cardboard to make the instruments easier to handle. Practise using your clinometer and transit as you proceed through the activities in this module.

You may be able to work with an actual transit and clinometer. Several businesses, such as construction firms, have surveyors on staff. You may be able to job-shadow this position. As well, if you attend an open-house event at one of the post-secondary technology institutes (for example, NAIT or SAIT), you may be able to get some hands-on experience with these tools.

Working through Activities 1 to 6 will help you gain the skills and concepts that you will need to complete the module project and the module assignment. As you work through the activities, continue exploring the topic. Keep notes, samples, and ideas in the project section of your mathematics binder. Feel free to discuss your project with your study partner. You will also find it helpful to have assistance in taking measurements. Just remember that the work you submit must be your own.



ACTIVITY 1



Indirect Measurement

Do you live on a farm or have you driven past farm yards? Are you impressed by the heights of gambrel-roofed barns and towering silos? How high are these structures? It would be difficult to measure these heights using direct measurement; that is, by using a measuring device such as a tape measure. Is there some other way of measuring the heights of tall buildings and structures?

Yes, you can use trigonometry to measure the height indirectly. You were introduced to trigonometry in Grade 9. The word *trigonometry* means “triangle measurement.” Using trigonometry, you can calculate the lengths of the sides of a triangle or the measures of the angles of a triangle, provided you know some of these angles and sides.

In this activity, you will review how to use trigonometry to determine lengths and angles indirectly.

The three basic trigonometric ratios are the **sine**, **cosine**, and **tangent** ratios.





You should be familiar with these ratios from previous mathematics courses; however, the following is a brief review.



Turn to page 322 of the textbook and read Tutorial 7.1, “Indirect Measurement.” In particular, study the diagrams on the page to review the sine, cosine, and tangent ratios.

I recall the sine, cosine, and tangent ratios using the memory aid SOH•CAH•TOA—pronounced “soak a toe.”



$$\sin A = \frac{\text{length of the side opposite } \angle A}{\text{length of the hypotenuse}}$$

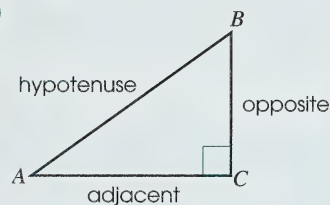
SOH

$$\cos A = \frac{\text{length of the side adjacent to } \angle A}{\text{length of the hypotenuse}}$$

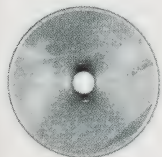
CAH

$$\tan A = \frac{\text{length of the side opposite } \angle A}{\text{length of the side adjacent to } \angle A}$$

TOA



For extra review on using sine, cosine, and tangent ratios, view the multimedia segment titled “Trig Ratios” on the companion CD.

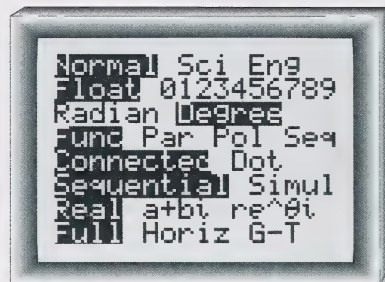


There are two main types of problems involving trigonometric ratios and a right triangle:

- You are given the measure of one of the acute angles and the length of a side, and you are expected to find the missing measures (sides and/or angles).
- You are given the lengths of two sides of the right triangle, and you are expected to find the missing measures.

When **solving a right triangle**—that is, finding the missing sides and angles—you will work with sine, cosine, and tangent on your graphing calculator.

Because angles will be given in degrees, set your calculator to degree mode. Press **MODE**; use the arrow keys to highlight “Degree” and press **ENTER**.



You are now ready to use the trigonometric function keys on your graphing calculator. You will use your graphing calculator to determine the trigonometric ratio for any angle between 0° and 90° as well as the angle from a given ratio.

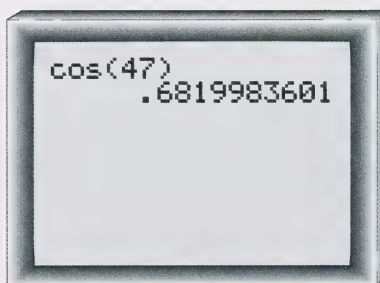
Example

Using a graphing calculator, find $\cos 47^\circ$. Round the answer to 4 decimal places. (**Note:** By convention, trigonometric ratios are rounded to 4 decimal places.)

Solution

Make sure the calculator is in degree mode. Press the following keys:

COS **4** **7** **)** **ENTER**



Pressing **COS** or any other trigonometric function pastes an opening parenthesis on the screen. Get in the habit of adding a closing parenthesis.

$\cos 47^\circ$ is about 0.6820.

When you are given a trigonometric ratio, you can use the **inverse trigonometric functions** to find the angle measure. (**Note:** The manual for the TI-83 calculator uses the terms *arcsine*, *arccosine*, and *arctangent* for the inverse functions.)

- Press $\boxed{2\text{nd}} \boxed{[\sin^{-1}]}$ to find the inverse function of sine.
- Press $\boxed{2\text{nd}} \boxed{[\cos^{-1}]}$ to find the inverse function of cosine.
- Press $\boxed{2\text{nd}} \boxed{[\tan^{-1}]}$ to find the inverse function of tangent.

Work through the following example to see how the inverse trigonometric function may be used.

Example

Using a graphing calculator, find the measure of acute angle A , such that $\tan A = 0.3456$. Round the answer to the nearest degree.

Solution

Use the inverse function of tangent to find the angle measurement from the given ratio.

Press the following keys:

$\boxed{2\text{nd}} \boxed{[\tan^{-1}]} \boxed{0} \boxed{\cdot} \boxed{3} \boxed{4} \boxed{5} \boxed{6} \boxed{)} \boxed{\text{ENTER}}$

The calculator will display the angle measurement.

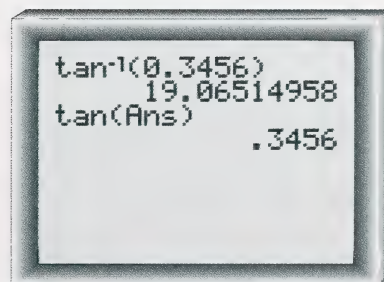
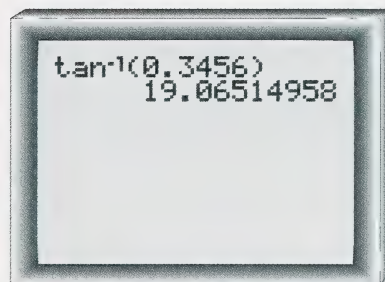
Therefore, the measure of $\angle A$ is about 19° .

Note: You can confirm that $\boxed{\text{TAN}}$

and $\boxed{[\tan^{-1}]}$ are inverses by

pressing $\boxed{\text{TAN}} \boxed{2\text{nd}} \boxed{[\text{ANS}]} \boxed{)} \boxed{\text{ENTER}}$.

You get the original ratio.



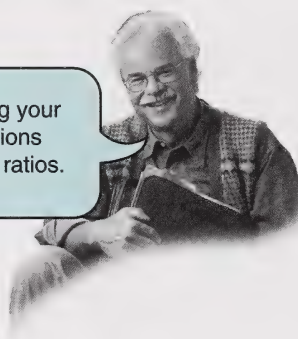
Before calculators and computers were available, tables were used for trigonometric values. A Greek mathematician by the name of Hipparchus, who is considered to be the father of trigonometry, developed the first trigonometric tables in 140 B.C. using chords of circles. In the second century, Ptolemy of Alexandria also worked with chords and developed tables accurate to five and six figures.

If you have access to the Internet, you can learn more about these mathematicians at the following website:

<http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/>



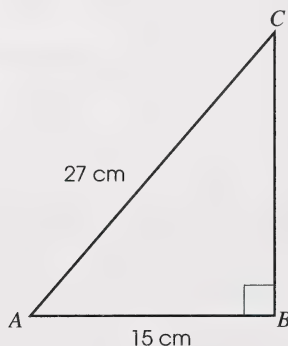
You will now practise using your calculator to solve questions involving the trigonometric ratios.



Work through the following example. Remember, when you are doing calculations in a multi-step problem, it is important to keep the calculator running.

Example

In right triangle ABC , determine $\cos A$, the measure of $\angle A$, and the length of side BC .

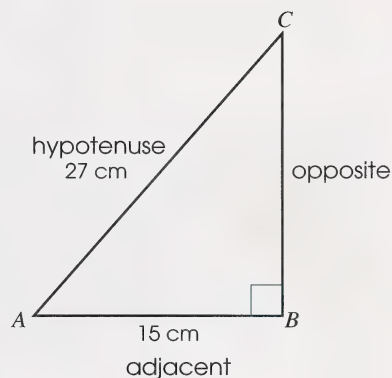


Note: Round the answer for the trigonometric ratio to four decimal places, the angle to the nearest degree, and the length of the side to the nearest centimetre.

Solution

In the diagram, side AB is the adjacent side since it lies next to $\angle A$; side AB is 15 cm. The longest side, AC , is the hypotenuse; it is 27 cm. The opposite side is BC .

Draw and label a diagram to clarify the situation.

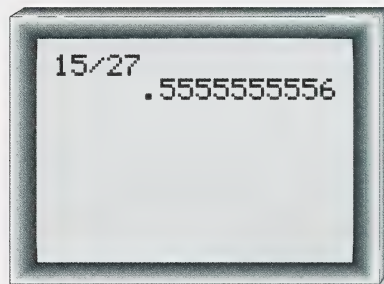


Step 1: Determine the trigonometric ratio of $\cos A$.

$$\begin{aligned}\cos A &= \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}} \\ &= \frac{AB}{AC} \\ &= \frac{15}{27} \\ &\div 0.5556\end{aligned}$$

The ratio for the cosine of $\angle A$ is about 0.5556.

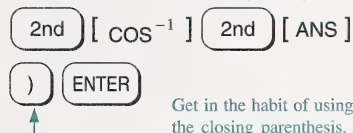
Do **not** clear your calculator. Keep this answer to use in the next step.



Step 2: Determine the measure of $\angle A$.

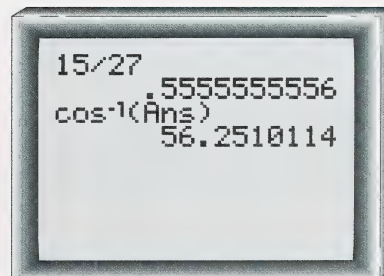
Determine the measure of the angle, given the cosine ratio, by using the inverse function of cosine.

These keys use the previous answer in the calculations.



The measure of $\angle A$ is about 56° .

Keep your calculator running.



Step 3: Determine the length of side BC .

Side BC is the side opposite to $\angle A$, so use the sine ratio to determine its length.

$$\sin A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$$

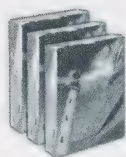
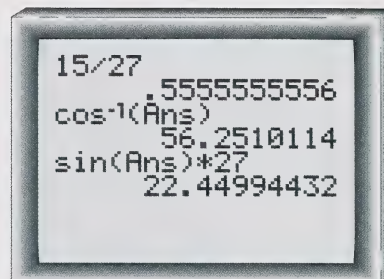
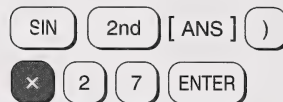
$$\sin A = \frac{BC}{AC}$$

$$BC = \sin A \times AC$$
$$\doteq 22.4$$

The length of BC is
about 22 cm.

Therefore, in right triangle ABC ,
 $\cos A \doteq 0.5556$, $\angle A \doteq 56^\circ$, and
 $BC \doteq 22$ cm.

Now you may clear
your calculator.



1. Turn to page 323 of the textbook and answer exercises 1 to 3 of Tutorial 7.1, “Indirect Measurement.” **Note:** You will need a calculator to determine the angle from the calculated ratio. Round the calculated ratios to four decimal places and the angles to the nearest degree.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 60–64.

Surveyors rely heavily on trigonometry for measuring distances that cannot be measured directly.

Before you study an example and do some exercises involving indirect measurement, you will do an investigation. You may choose to do this investigation with a partner.

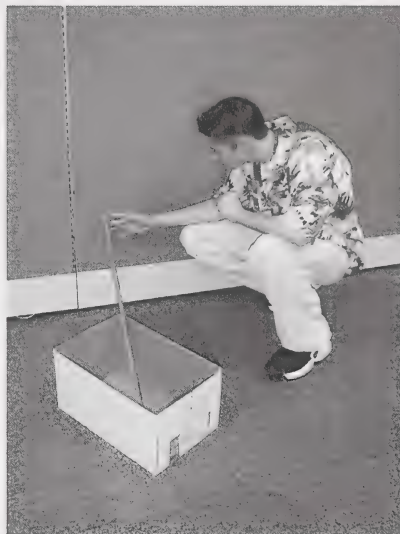




2. Turn to page 324 of the textbook and answer exercises 1 to 8 of “Investigation 1: Indirect Measurement.” **Note:** You will need a cardboard box about 20 cm to 30 cm high and a metre-stick. Round your measurements to the nearest 0.1 cm.



Determining Angle 1



Determining Angle 2

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 64–68.

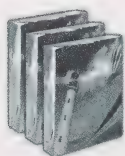
Now that you have had some experience in using the sine and tangent ratios with triangles, you should be ready to look at an example of using trigonometry in problem solving. Then, you will solve some problems involving the basic trigonometric ratios.



Distances that cannot be measured directly include the distance across a lake, the height of a kite, and the height of a tall building.

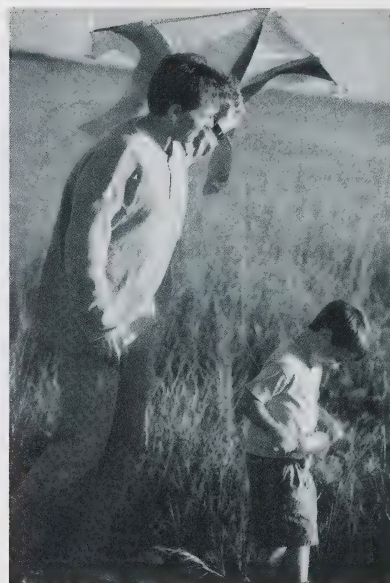
Problems involving indirect measurement and surveying often use terms such as **angle of depression**, **angle of elevation**, and **alternate angles**. You should have encountered these terms in a previous mathematics course. For a review of these terms, watch the multimedia segment titled “Angles” on the companion CD.





Turn to pages 325 and 326 of the textbook and read through “Example 1: Determine the height of a kite.”

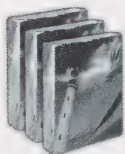
3. Turn to page 326 of the textbook and answer exercises 1 and 2 of “Discussing the Ideas.”
4. Turn to pages 326 and 327 of the textbook and answer exercises 1, 2, 5, and 6 of “Exercises: Checking Your Skills.”
5. Turn to page 327 of the textbook and answer exercise 7 of “Exercises: Extending Your Thinking.”



Compare your responses with the suggested answers in the Appendix, Activity 1, pages 68–72.

LOOKING BACK

In this activity, you used indirect measurement to determine the height of a building or the distance across a river or lake. You also used a calculator to determine the trigonometric ratio of an angle as well as the angle, given the trigonometric ratio. You used inverse trigonometric functions. You then used your new knowledge to solve problems involving indirect measurement.



6. Turn to page 327 of the textbook and answer the question in “Communicating the Ideas.”

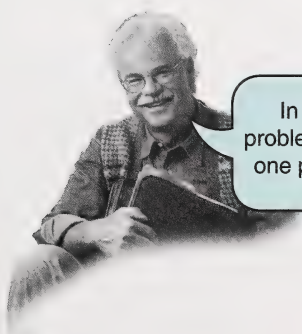
Compare your response with the suggested answer in the Appendix, Activity 1, page 73.



Solving Problems Using More Than One Right Triangle

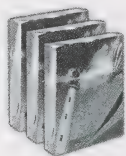
Big-city skylines are filled with the silhouettes of towering buildings and, often, the outlines of giant cranes.

Sometimes you may know the height of one tall building and you want to determine the height of another, nearby highrise. To determine the height of the second highrise, you need to use more than one right-angle triangle. In this case, the right-angle triangles are in the same plane. Some problems involving more than one right-angle triangle involve right-angle triangles in different planes.



In this activity, you will solve problems involving right triangles in one plane and in different planes.

SOLVING PROBLEMS USING RIGHT TRIANGLES IN ONE PLANE



Two right-angle triangles in one plane can be used to compare the heights of two trees by measuring along the ground or the height of two highrise towers by measuring vertically.

Turn to pages 332 to 334 of the textbook and study Example 1 and Example 2. Notice that the angle of depression in the diagram at the top of page 334 is incorrectly given as 30° . The angle should be 34° , as it is given in the problem on page 333 and as it is used in the solution on page 334.

Answer the following questions as you read Example 1 and Example 2.

1. Are the two triangles in the diagram near the bottom of page 332 in the same plane?
2. Do you think it is necessary that point C be halfway between the two trees?
3. Explain why the tangent ratio is used to determine the heights of the trees.
4. Are the triangles in the diagram at the top of page 334 in the same plane?
5. Why is the lower triangle used to determine the length of x in Example 2?

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 73–74.

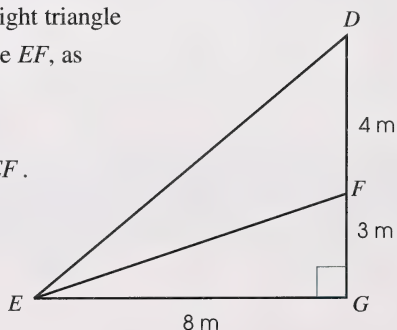
Often, when you solve problems involving two right triangles, you need to solve each triangle separately, and then add the two results.



Example

A plot of land in the shape of right triangle DEG is to be subdivided by line EF , as shown in the diagram.

Calculate the measure of $\angle DEF$.
Round the answer to the nearest degree.

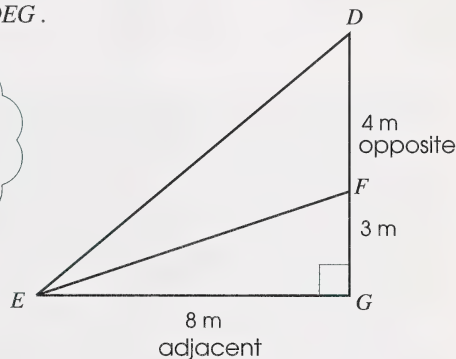


Solution

To solve any trigonometric problem, you need to make a plan. Notice that $\triangle DEF$ is not a right triangle. To find the measure of $\angle DEF$, you first need to find the measures of $\angle DEG$ and $\angle FEG$. Then, subtract the measure of $\angle FEG$ from the measure of $\angle DEG$. To avoid rounding errors, round $\angle DEG$ and $\angle FEG$ to nearest tenth of a degree.

Step 1: Find the measure of $\angle DEG$.

$$\begin{aligned} DG &= DF + FG \\ &= 4 + 3 \\ &= 7 \end{aligned}$$



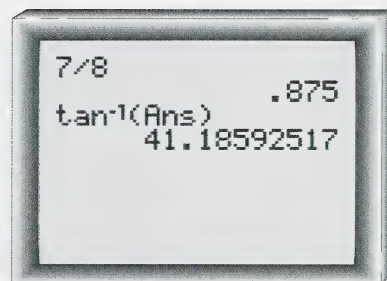
The trigonometric ratio of the length of the opposite side, DG , to the length of the adjacent side, EG , is the tangent of $\angle DEG$.

$$\begin{aligned} \tan \angle DEG &= \frac{\text{length of opposite side}}{\text{length of adjacent side}} \\ &= \frac{DG}{EG} \\ &= \frac{7}{8} \\ &= 0.8750 \end{aligned}$$

Do **not** clear your calculator. Use the unrounded answer in further calculations.

Use the inverse trigonometric function of tangent to find the measure of $\angle DEG$.

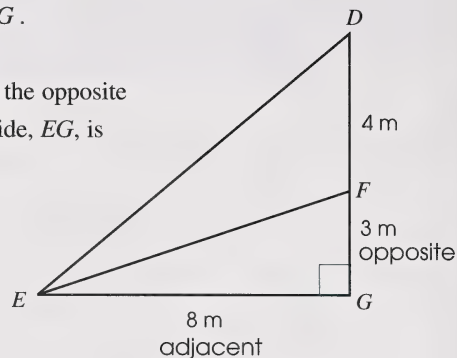
$$\angle DEG \doteq 41.2^\circ$$



Recall, to avoid rounding errors, round the angles to nearest tenth of a degree.

Step 2: Find the measure of $\angle FEG$.

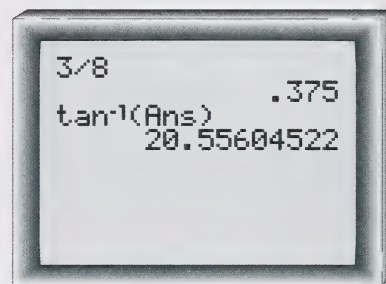
The trigonometric ratio of the opposite side, FG , to the adjacent side, EG , is the tangent of $\angle FEG$.



$$\begin{aligned}\tan \angle FEG &= \frac{\text{length of opposite side}}{\text{length of adjacent side}} \\ &= \frac{FG}{EG} \\ &= \frac{3}{8} \\ &= 0.3750\end{aligned}$$

Use the inverse trigonometric function of tangent to find the measure of $\angle FEG$.

$$\angle FEG \doteq 20.6^\circ$$



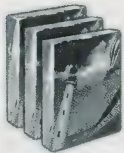
Step 3: Determine the measure of $\angle DEF$. Subtract the measure of $\angle FEG$ from the measure of $\angle DEG$.

$$\begin{aligned}\angle DEF &= \angle DEG - \angle FEG \\ &\doteq 41.2^\circ - 20.6^\circ \\ &\doteq 20.6^\circ\end{aligned}$$

The measure of $\angle DEF$ is about 21° .

Now that you have analysed some examples involving more than one right triangle, you should be ready to try some exercises. Be sure to draw a detailed diagram where necessary and write a plan as to how you will solve each problem.





6. Turn to pages 337 to 339 of the textbook and answer exercises 1, 2, 3, 5, 7, and 8 of “Exercises: Checking Your Skills.”

Note: Keep your calculator running in exercises having more than one part. Do **not** clear the calculator between parts. Round the lengths in the answers for exercises 1, 7, and 8 to the nearest whole number.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 74–82.

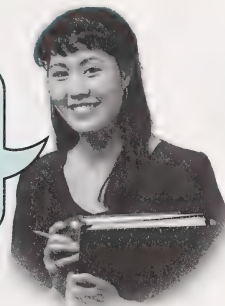
There are several examples of suspension bridges around the world.

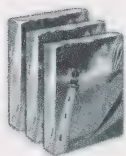


If you have access to the Internet, you may wish to view the Humber Bridge. Several sites provide images of this spectacular suspension bridge.

SOLVING PROBLEMS USING RIGHT TRIANGLES IN DIFFERENT PLANES

So far, you have solved problems involving two right triangles in the same plane. Some trigonometric problems involve two right triangles in different planes.





Turn to pages 335 and 336 of the textbook and study “Example 3: Using right triangles in different planes.” Then answer the questions that follow.

7. a. Why do the two right triangles in the diagram at the top of page 335 of the textbook need to be at right angles?

b. Name the common side to $\triangle TBR$ and $\triangle SRB$.
8. Turn to page 336 of the textbook and answer exercises 1, 2, and 3 of “Discussing the Ideas.”
9. Turn to page 339 of the textbook and answer exercise 9 of “Exercises: Checking Your Skills.”

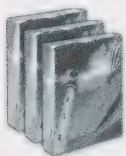
Note: Write a plan explaining how you will calculate the height of the cliff. Make appropriate drawings. Round your answer to the nearest metre.



Compare your responses with the suggested answers in the Appendix, Activity 2, pages 82–84.

LOOKING BACK

In this activity, you solved problems involving more than one right triangle. You differentiated between calculations with right triangles in the same plane and in different planes. You also identified the importance of drawing a diagram and writing a plan to help solve each problem.



10. Turn to page 340 of the textbook and answer the question in “Communicating the Ideas.”

Compare your response with the suggested answer in the Appendix, Activity 2, page 84.

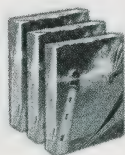


ACTIVITY 3

Sine and Cosine for Angles from 0° to 180°

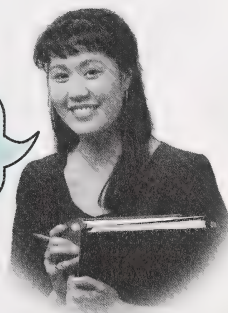
You are watching two sailboats at the far end of a large lake. You determine the angle to be 108° between the two lines of sight to the boats. An angle greater than 90° is called an **obtuse angle**.

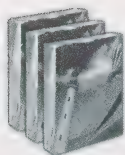
Up to this point, you have used your graphing calculator to determine the trigonometric ratios of **acute angles**. Now you will use your graphing calculator to determine the sine and cosine ratios of angles between 0° and 180° .



Turn to page 341 of the textbook and read Tutorial 7.3, “Extending the Concepts of Sine and Cosine for Angles from 0° to 180° .” Press **MODE** on your calculator to check whether your calculator is in degree mode.

You will now use your calculator to determine the sine of any angle between 0° and 180° .





1. Turn to pages 341 and 342 of the textbook and answer exercises 1 to 8 of “Investigation 1: Extend the Concept of Sine for Angles between 0° and 180° .”

Note: For textbook exercise 6, follow these additional steps and round answers to four decimal places:

Step 1: Use the arrow keys to move the cursor along the line on the graph.

Step 2: Move the cursor until the given x -value appears in the lower left corner of the window. In column 2 of the table in your notebook, write the y -value that appears in the lower right corner of the screen. (The x -value is the measure of $\angle x$; the y -value is the sine ratio of $\angle x$.)

Step 3: Move the cursor to find another angle with the same y -value as in Step 2.

Hint: Move the cursor to the far right of the graph. In column 3 of your table, write the angle with this same sine value.

Step 4: Repeat Steps 2 and 3 for the remaining values of angle x .

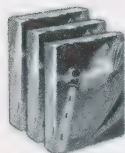
Compare your responses with the suggested answers in the Appendix, Activity 3, pages 85–87.



I discovered that there are two angles between 0° and 180° that have the same value for the sine ratio. The sum of the two angles is always 180° .



That's right. In the next investigation, you will explore the cosine ratio for angles between 0° and 180° .



2. Turn to page 342 of the textbook and answer exercise 1 of “Investigation 2: Extend the Concept of Cosine for Angles between 0° and 180° .” **Note:** Follow the same steps for exercise 6 as in question 1. Round answers to four decimal places.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 87–89.

In Investigation 1 and Investigation 2, you analysed the sine and cosine values for angles between 0° and 180° . You saw that an angle less than 90° had a related angle greater than 90° . These pairs of related angles are called **supplementary angles**. If you look at the graphs of the sine and cosine of the angles between 0° and 180° , you can see that this relation arises from the symmetry of the graph.



3. Turn to page 343 of the textbook and answer exercises 1, 3, 4, and 5 of “Discussing the Ideas.”

Compare your responses with the suggested answers in the Appendix, Activity 3, page 89.

Using the relations that you have identified in the previous exercises, you can answer a number of questions involving sine and cosine values without the use of a calculator.

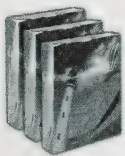
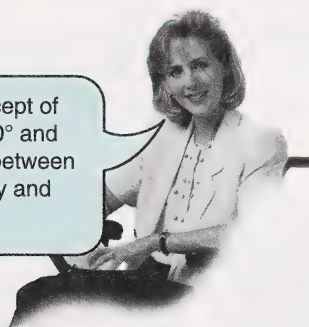


4. Turn to page 343 of the textbook and answer exercises 1, 2, and 4 of “Exercises: Checking Your Skills.”
5. Turn to pages 343 and 344 of the textbook and answer exercises 3.a. to 3.f., 5.a., 5.f., 5.g., and 5.i. of “Exercises: Checking Your Skills.” **Note:** Use your calculator where necessary to answer the questions. You may need to use trial and error.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 90–91.

LOOKING BACK

In this activity, you extended the concept of sine and cosine to angles between 90° and 180° . You discovered the relationship between the sine and cosine of supplementary and complementary angles.



6. Turn to page 345 of the textbook and answer the question in “Communicating the Ideas.”

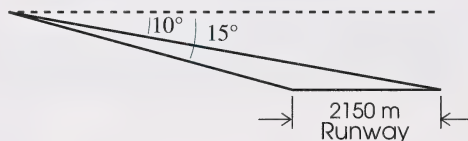
Compare your response with the suggested answer in the Appendix, Activity 3, page 91.

ACTIVITY 4



The Sine Law

An airplane is coming in to land on a runway that has a length of 2150 m. At one point in the descent, the angle of depression to the far end of the runway is 10° and to the near end of the runway it is 15° . How far is the airplane from the near end of the runway?

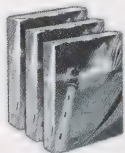


The triangle formed by the runway and the lines from each end of the runway to the airplane is not a right angle triangle. To solve this problem, you need to be able to solve an **oblique triangle**.

In this activity, you will solve oblique triangles given two angles and a side **or** two sides and an angle opposite one of the sides.

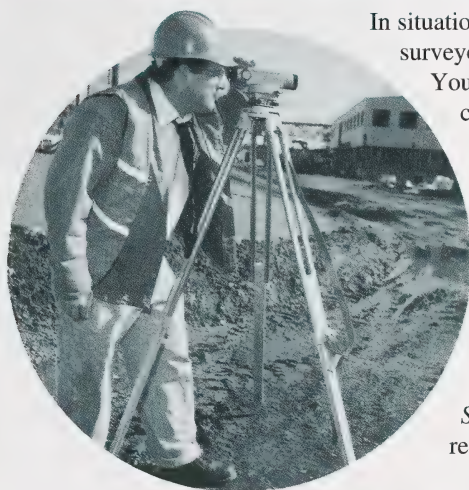
Turn to pages 346 and 347 of the textbook and read the introduction of Tutorial 7.4, “The Sine Law.” Then, answer the following questions based on the reading.

1. What assumption is made about the direction of AB relative to the bank of the river?
2. Is the distance AB the shortest distance across the river?
3. Why is the bridge to be built at these points?



4. What type of triangle is $\triangle ABC$?
5. How are the angles and sides named in the oblique triangle shown on page 347?

Compare your responses with the suggested answers in the Appendix, Activity 4, page 91.



In situations such as the one in the tutorial, a surveyor uses a **transit** to measure the angles.

You may want to practise using the transit you constructed at the start of this module.

Follow the instructions on page 329 of your textbook and in the Extra Help of this Student Module Booklet.

The relationships among the angles and sides of an oblique triangle are described by the Sine Law. You can use a ruler and protractor or a computer and a geometry package (for example, *The Geometer's Sketchpad*®) to investigate these relationships. You may wish to do both.

You may use the Internet to explore various geometry software packages. Some sites offer demo downloads.

6. Question 6.b. is optional.

- a. Turn to pages 347 and 348 in the textbook and answer exercises 1 to 5 of "Investigation 1: The Sine Law." Follow the given directions very carefully.
- b. If you have access to a computer with a dynamic geometry package, such as *The Geometer's Sketchpad*®, follow the instructions in "Investigation 2: The Sine Law Using Technology," on pages 348 and 349 of the textbook. Then, answer exercises 1 to 5.

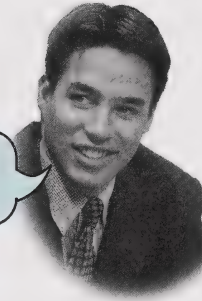
Compare your responses with the suggested answers in the Appendix, Activity 4, pages 92–93.

In doing the investigation(s), you should have found the following relationships to be true for the sides and opposite angles in any triangle.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{and} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

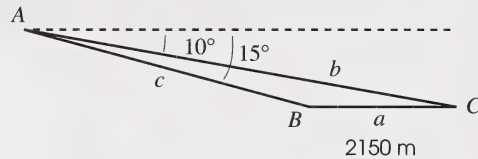
The equalities you identified are called the **Sine Law**. The Sine Law can be used to solve various types of problems by using any pair of these ratios according to the given sides and angles.

Work through the following example.



Example

An airplane is landing on a runway that has length of 2150 m. At one point in the descent, the angle of depression to the far end of the runway is 10° and to the near end of the runway it is 15° . How far is the airplane from the near end of the runway? (This is the problem given at the beginning of Activity 4.) **Note:** Round your answer to the nearest metre.



Solution

Make a plan to solve the problem. The distance of the plane from the near end of the runway is the length of side AB , or c . You are given side a . To find the measure of $\angle A$, find the difference between the angles of depression. Then, find the measure of $\angle C$. Angle C is an alternate angle to the angle of depression of 10° —the angle to the far end of the runway. Because you now know the length of a side and the measures of the two adjacent angles, you can use the Sine Law to find side c .

Step 1: Find the measure of $\angle A$ in $\triangle ABC$ by finding the difference between the angles of depression.

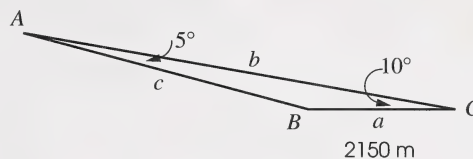
$$\begin{aligned}\angle A &= 15^\circ - 10^\circ \\ &= 5^\circ\end{aligned}$$

The measure of $\angle A$ is 5° .

Step 2: Find the measure of $\angle C$ in $\triangle ABC$.

The measure of $\angle C$ is 10° (alternate angles).

Step 3: Draw and label a diagram with the new information.



Step 4: Use the Sine Law to find c .

Identify the equation (the pair of ratios) needed to find c , where the other three values are known.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{c}{\sin 10^\circ} = \frac{2150}{\sin 5^\circ}$$

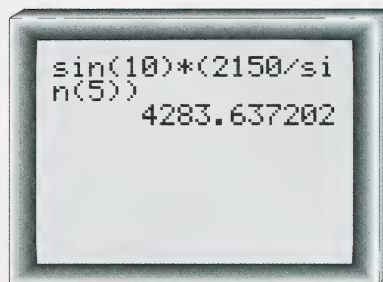
← Substitute the known values.

$$c = \sin 10^\circ \left(\frac{2150}{\sin 5^\circ} \right)$$

← Solve for c .

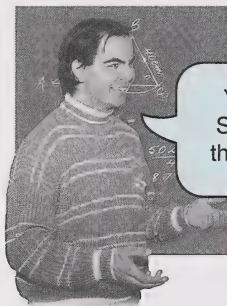
$$\doteq 4283.6$$

Therefore, to the nearest metre, the airplane is about 4284 m from the near end of the runway.

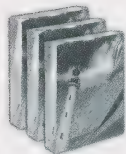


7. Turn to pages 349 and 350 in the textbook and answer exercises 1 to 5 of “Investigation 3: Using the Sine Law.” This investigation solves the problem posed at the start of Tutorial 7.4, “The Sine Law” (page 346 in the textbook).

Compare your responses with the suggested answers in the Appendix, Activity 4, page 93.



You now have some knowledge of the Sine Law. For a better understanding of the Sine Law, do the following exercises.



8. Turn to page 350 of the textbook and answer exercises 1, 3, and 5 of “Discussing the Ideas.”
9. Turn to page 355 of the textbook and answer exercises 1 and 2.a. of “Exercises: Checking Your Skills.” Determine the lengths of the sides to the nearest tenth and the measure of the angles to the nearest degree.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 94–95.

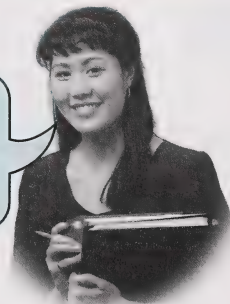
There are many interesting problems that can be solved using the Sine Law. When solving problems, read the problem carefully, draw and label a diagram to illustrate the situation, and make note of all given and required measurements.

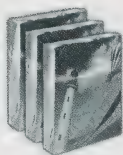
Turn to pages 351 to 354 of the textbook and study Examples 1 to 3; then, answer the following questions.

10. Turn to page 355 of the textbook and answer exercises 1 and 2 of “Discussing the Ideas.”
11. Turn to page 354 of the textbook and reread the solution of “Example 3: Determine an angle in an obtuse triangle.” Notice that the required angle is in the numerator position of the ratio. Why is this form of the Sine Law used?

Compare your responses with the suggested answers in the Appendix, Activity 4, page 95.

Now you can try doing some real-life problems that can be solved using the Sine Law.





12. Turn to pages 356 to 358 of the textbook and answer exercises 4, 6, 7, and 8 of “Exercises: Checking Your Skills.” **Note:** The illustration for textbook exercise 7 is on page 357 (not page 358 as stated in the textbook).

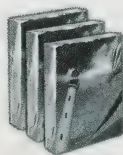


Compare your responses with the suggested answers in the Appendix, Activity 4, pages 96–100.

LOOKING BACK



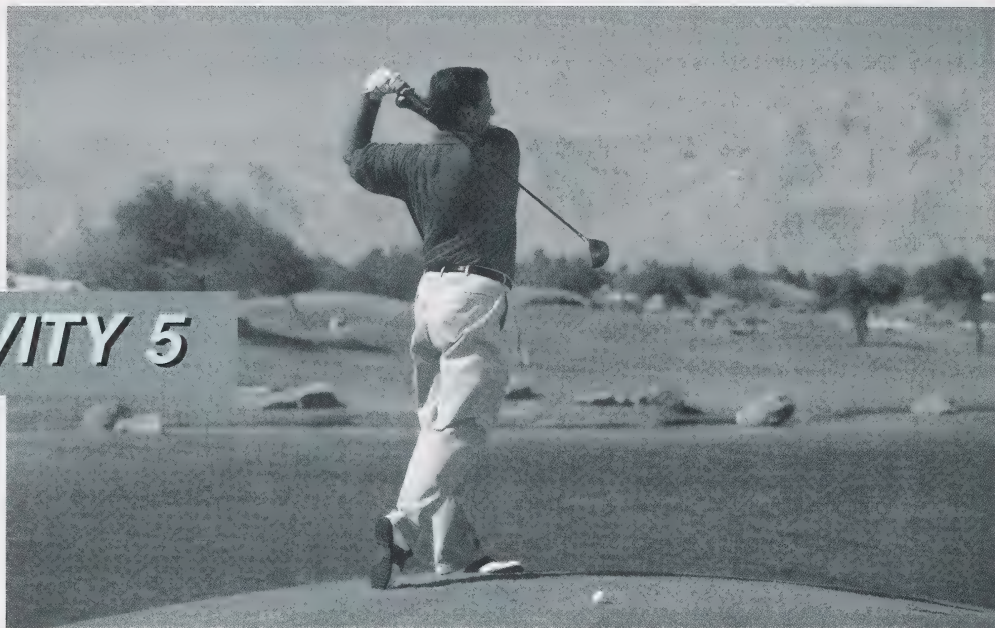
In this activity, you explored the Sine Law and completed exercises and problems involving the Sine Law. You will use the Sine Law to complete the module project.



13. Turn to page 350 of the textbook and answer the question in “Communicating the Ideas.”

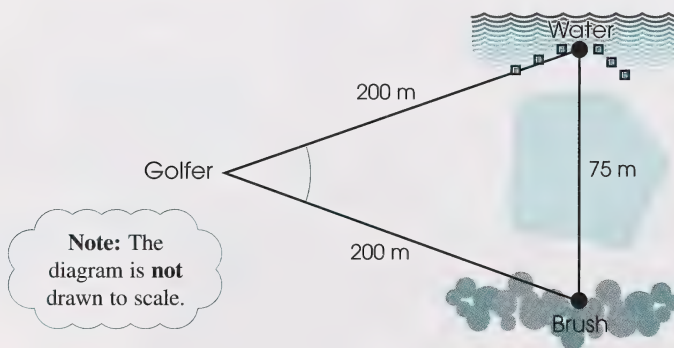
Compare your response with the suggested answer in the Appendix, Activity 4, page 100.

ACTIVITY 5



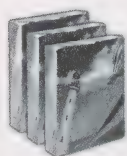
The Cosine Law

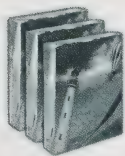
Many golfers find accuracy a problem when hitting a golf ball. Suppose you must aim down a fairway 75 m in width, with water on the left and thick brush on the right. If you can drive a golf ball 200 m, within what angle can you drive the ball safely?



In this activity, you will use the **law of cosines** (also called the **Cosine Law**) to solve oblique triangles, such as the one created with the golf shot, for unknown angles when given the lengths of all three sides. The law of cosines can also be used to solve an oblique triangle when you are given two sides and the angle between those sides.

Turn to pages 360 to 362 of the textbook and study Examples 1 and 2 of Tutorial 7.6, “The Cosine Law.” Example 1 and Example 2 use the Cosine Law to find the third side of an oblique triangle, given two sides and the included angle.

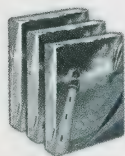




1. Turn to page 364 of the textbook and answer exercises 1 to 5 of “Discussing the Ideas.”
2. Turn to page 365 of the textbook and answer exercise 1 of “Exercises: Checking Your Skills.”

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 101–102.

In the previous exercise, you used the Cosine Law to find the third side of an oblique triangle. You can also use the Cosine Law to find the angles in an oblique triangle, given the three sides.



Turn to pages 363 and 364 of the textbook and study “Example 3: Use the Cosine Law to determine an angle of a triangle.”

3. Turn to page 365 of the textbook and answer exercise 2 of “Exercises: Checking Your Skills.”

Note: Round your answers to the nearest tenth of a degree.



Compare your responses with the suggested answers in the Appendix, Activity 5, page 103.

Solving problems involving the Cosine Law uses similar techniques to problem solving with the Sine Law. Read the problem carefully, draw and label a diagram to illustrate the situation, make note of all given and required measurements, and make a plan.

Example

Two ships leave port at the same time. One sails at 19 km/h on a bearing of 192° . The other sails at 14 km/h on a bearing of 233° . How far apart are the ships after 3 h? Express your answer to the nearest tenth of a kilometre.

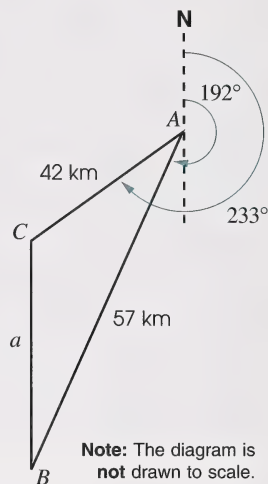
Solution

Step 1: Draw a labelled diagram to illustrate the problem. Clarify the problem.

The angle between the paths of the two ships, $\angle A$, is equal to the difference of the two bearings.

The ship on a bearing of 192° will travel $19 \times 3 = 57$ km in 3 h.

The ship on a bearing of 233° will travel $14 \times 3 = 42$ km in 3 h. Distance a is the distance between the two ships after 3 h.



Step 2: Make a plan to solve the problem. First, determine the measure of $\angle A$. You will then know two sides and the included angle. You can use the Cosine Law to find side a , which is the side opposite the included angle.

Step 3: Determine $\angle A$. The measure of $\angle A$ is the difference between the two bearings.

$$\begin{aligned}\angle A &= 233^\circ - 192^\circ \\ &= 41^\circ\end{aligned}$$

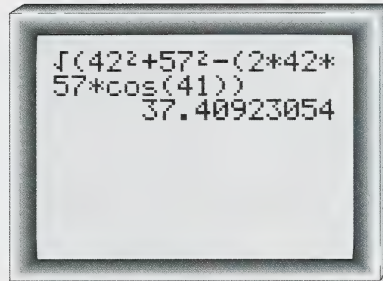
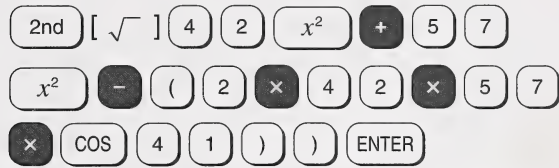
The measure of $\angle A$ is 41° .

Step 4: Use the Cosine Law to determine the length of a .

Choose the form of the Cosine Law that will fit the situation in the problem.

$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cos A \\ a^2 &= 42^2 + 57^2 - 2(42)(57) \cos 41^\circ && \leftarrow \text{Substitute known values.} \\ a &\doteq 37.41\end{aligned}$$

Following is the calculator-key sequence used:



The two ships are about 37.4 km apart after 3 h.

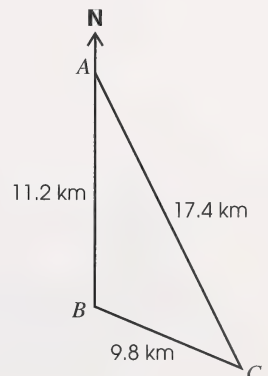
Solve the following problems involving the Cosine Law.



4. Turn to pages 365 and 367 of the textbook and answer exercises 3 and 10 of "Exercises: Checking Your Skills."
5. A hiking trail is marked out as shown in the diagram. The mapmaker wants to supply bearings for the leaflet that describes the trail.

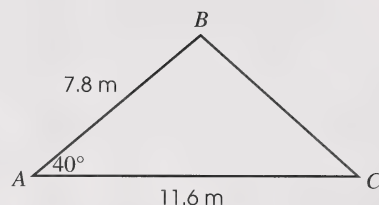
Calculate the measures of the angles between the paths and determine the bearing for each trail. Express the measures of the angles to one decimal place; express the bearing using three digits.

Note: Using bearings is discussed on page 357 of the textbook in exercise 7. You worked with bearings in Activity 4.



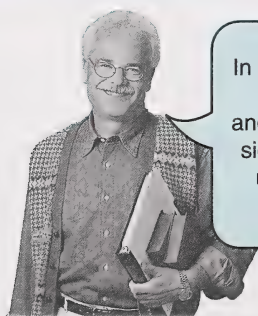
6. Roofs can be built using various specifications.

A homeowner has a roof designed as shown in the diagram. Calculate the length of the steepest roof truss (side BC) and the measures of the unknown angles. Round the length to one decimal place; round the angles to the nearest degree.

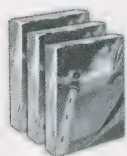


Compare your responses with the suggested answers in the Appendix, Activity 5, pages 104–109.

LOOKING BACK



In this activity, you were introduced to the Cosine Law. You used the Cosine Law to find the sides and/or angles in oblique triangles in which you were given two sides and the included angle or three sides. You also used the Cosine Law to solve real-world problems involving oblique triangles.



7. Turn to page 367 of the textbook and answer the question in “Communicating the Ideas.”

Compare your response with the suggested answer in the Appendix, Activity 5, page 109.

ACTIVITY 6

Selecting a Strategy for Problem Solving

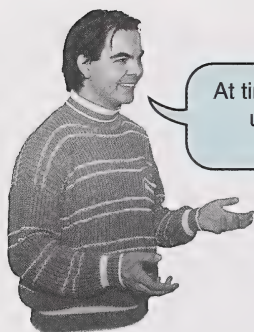
There are many problems that involve solving oblique triangles.

In the previous two activities, you solved problems using the Sine Law and the Cosine Law with oblique triangles. Solving many problems involving trigonometry requires both oblique-triangle laws (Sine Law and Cosine Law) and basic, right-triangle ratios (sine, cosine, and tangent). The strategy for finding a solution requires understanding the problem and then selecting the appropriate combination of laws.

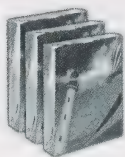
Turn to pages 368 to 370 of the textbook and study “Example 1: Solve a problem using a combination of oblique and right triangle trigonometry.”

1. Explain the following terms in your own words:
 - a. trigonometry
 - b. right-triangle trigonometry
 - c. oblique-triangle trigonometry
 - d. the angle sum property
2. How can you find the third angle in a triangle if you know the measures of two of the angles in the triangle?

Compare your responses with the suggested answers in the Appendix, Activity 6, page 109.



At times, a combination of laws must be used to solve problems involving oblique triangles.



3. Turn to pages 370 and 371 of the textbook and study “Example 2: Using a combination of laws to solve an oblique triangle.”

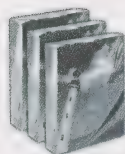
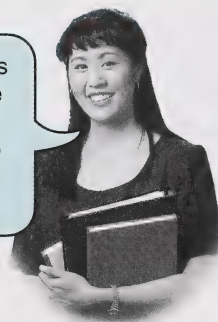
List the steps involved in solving the problem in Example 2.

4. Turn to page 372 of the textbook and answer exercises 1, 2, and 4 of “Discussing the Ideas.”
5. Turn to pages 372 to 374 of the textbook and answer exercises 2, 4, and 7 of “Exercises: Checking Your Skills.”

Compare your responses with the suggested answers in the Appendix, Activity 6, pages 109–112.

LOOKING BACK

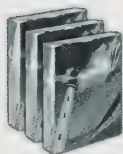
In this activity, you solved problems involving the Sine Law, the Cosine Law, and the basic trigonometric ratios. You discovered the need to plan your approach to solving a problem.



6. Turn to page 375 of the textbook and answer the question in “Communicating the Ideas.”

Compare your response with the suggested answer in the Appendix, Activity 6, page 113.

Follow-up Activities



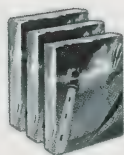
Review the skills and concepts and the important results and formulas in this module by reading the summary in “What Do I Need To Know?” on page 378 of the textbook.

Multi-step problems require that you draw a well-labelled diagram and plan which parts of the problem need to be answered first.

1. Turn to page 379 of the textbook and answer exercises 1 to 7 of Part A of “What Should I Be Able To Do?” This series of questions should help you understand the sequence and planning required to answer a multi-step problem.



Compare your responses with the suggested answers in the Appendix, Follow-up Activities, pages 113–116.



For more practice with problem solving, do the following exercises.

2. Turn to page 380 of the textbook and answer exercises 8, 10, 11, and 12 of Part B of “What Should I Be Able To Do?”

Compare your responses with the suggested answers in the Appendix, Follow-up Activities, pages 116–118.

If you had difficulties understanding the skills and concepts in Module 7: Trigonometry, it is recommended that you do the Extra Help. If you have a clear understanding of the skills and concepts in this module, it is recommended that you do the Enrichment. You may decide to do both.

EXTRA HELP

In this module, you read about various situations using trigonometry to solve problems involving angles and sides of right-angle and oblique triangles. You found out how surveyors use special instruments to measure angles of elevation and angles between the sides of a parcel of land. You built your own measuring devices to determine angles of elevation and to survey a parcel of land.

The extra help offers hints that you may find useful in constructing and using a clinometer and a transit.

Constructing and Using a Clinometer

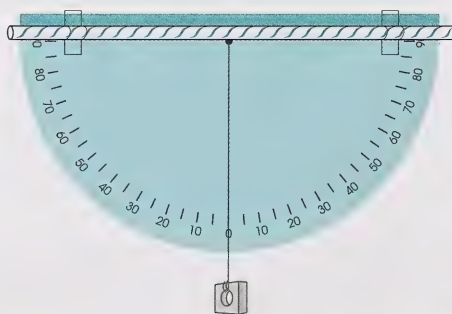
Materials

- a thin drinking straw or pencil
- a piece of thread or string, about 20 cm long
- a weight (such as a nut or washer)
- adhesive tape and glue
- the cutout of the 180° protractor (located in the back of the Appendix)
- a piece of cardboard (about 28 cm $\times$ 21 cm)

Construction

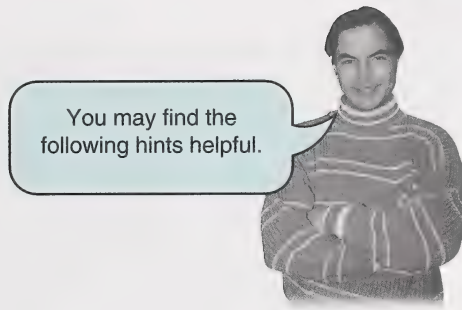
Follow these directions:

- Glue the 180° protractor located at the back of the Appendix onto a piece of cardboard of the same size. Then, cut out the protractor.
- Tape a drinking straw as indicated on the diagram.



- Tie a nut (or other suitable weight) to a thread. Run the free end of the thread through the centre point and tape it securely with adhesive tape on the back of the cardboard. The weighted end of the thread should hang down through 0° and below the cardboard.

Using a Clinometer



- Point the straw at the top of the object you wish to measure. Look along the straw and make sure it is aligned with the top of the object as closely as possible.



- Have someone read the measure of the angle that the string is aligned with.

Note: If you have to do the reading yourself, hold the string with your thumb once you have the clinometer properly aligned; then read the angle.

- The reading gives you the angle of elevation or depression from a horizontal line at your eye level. Remember to add your height to the height you obtain in your calculations.

Practise using your clinometer by finding the height of a nearby building (or other structure).

- Stand with your clinometer so that you can see the top of the building. (You may wish to mark this point using a coin, small stick, or other marker.)

- Use your clinometer to measure the angle of elevation (see the preceding instructions).



- Measure the distance from where you are standing (use the marker if you are taking the measurements on your own) to the base of the building or structure.

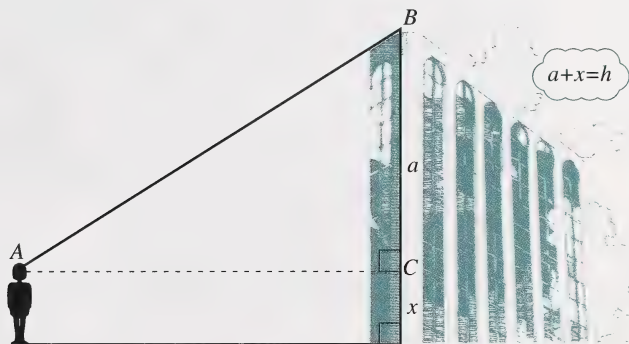


You may need a weight to hold down the tape measure at your marker if you are doing the measurement on your own.

It's helpful to have a person assist you in reading the clinometer and then using a tape measure to find the distance.



- Draw a diagram to illustrate your findings. Label the diagram to make calculations easier.



- Set up your equation using the correct trigonometric ratio.

$$\begin{aligned}\tan BAC &= \frac{\text{opposite side}}{\text{adjacent side}} \\ &= \frac{BC}{AC}\end{aligned}$$

- Rearrange the equation, substitute your values for AC and $\angle BAC$, and solve for a .

Don't forget to add your height, x , to the length of BC (side a) in order to find the height of the building.



1. Use the clinometer to find the height of a building or object for which you can find the actual height. Was your investigative result reasonable?

Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Extra Help, page 119.

Constructing and Using a Transit



You may find the following helpful.

Materials

- a drinking straw
- a needle or nail
- a string about 40 cm long
- a weight (such as a nut, washer, or paper clip)
- adhesive tape and glue
- the cutout of the 360° protractor (located in the back of the Appendix)
- a piece of cardboard (about $28\text{ cm} \times 21\text{ cm}$)
- doweling* (1 cm–2 cm in diameter and about 100 cm long)

* You could also use a strip of wood about $2\text{ cm} \times 1\text{ cm}$ and about 100 cm long or a (camera) tripod.

Construction

Follow these directions.

- Glue the 360° protractor from the back of the Appendix onto a piece of cardboard of approximately the same size. Cut out the protractor.
- Use a needle or nail to make a hole through the centre of the protractor. Thread the string from the back to the front, tie a knot in the string at the back of the cardboard, and tape the knot to the cardboard.
- Pass the string through the straw. Tie a weight to the free end of the string.
- Attach the centre of the protractor to the doweling, wood, or tripod. (You may need to make a small platform on the doweling and experiment to make the attachment secure and level. Hot glue provides a quick attachment. You may wish to use a reusable sticky-tack material on a tripod base.)

Using a Transit

- Pick a triangular or quadrilateral-shaped area to survey. Use the corner of a building, a tree, or a fence post as the corners of your area.



If you are in an open field or parking lot, use stakes or some other small objects to locate your corners. (You could tie pieces of bright plastic tape to the stakes to make them more visible.)

- Measure and record the distance from one corner to the next corner of your area.

It is helpful to have someone assist you in taking the measurements.

- Place your transit at the second location and line up the 0° with the peg at the first location.
- Locate your third corner with a peg or object and move the straw until you can site the peg or object at the third corner.

Read and record the angle of the third peg from the second peg.



- If you have a triangular area, measure the distance from the second peg to the third peg and you are done. Draw and label a diagram and use the Cosine Law to determine the measures of the remaining angle and side.

If you have a quadrilateral area, you will need to move the transit to the third peg and determine the angle to the fourth peg.

2. Practise using your transit to determine the angles of a triangular area marked by trees, the corner of a building, or some other object. Use your transit and measuring tape to determine the measures of two sides and an included angle.
 - a. Draw and label a diagram of your area.
 - b. Use trigonometry to determine the remaining side and angles (round your answers to the nearest metre and degree).

Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Extra Help, pages 119–120.

ENRICHMENT

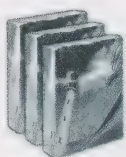
Footprints left in fresh snow, sand, or earth can reveal many details about what or who was passing by. Criminologists have used their special skills in reading footprints to help identify suspects in crimes. The study of footprints and tracks can be very exciting.

Turn to pages 363 and 364 of the textbook and review “Example 3: Use the Cosine Law to determine an angle of a triangle.” This example discusses the length of a race walker’s stride and pace.

1. Do the following exercise.

Step 1: Walk as you would normally in sand or snow and measure the lengths of your stride and pace. Use this information to determine your step angle.

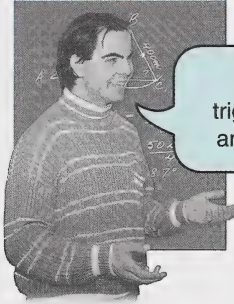
Step 2: Then, walk quickly—as if you were in a walking race in the sand or snow. Measure the lengths of your stride and pace. Use these measurements to determine your step angle.



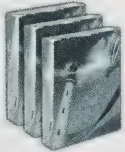
Step 3: Compare your step angle and your stride and pace in Step 1 to that in Step 2.

Step 4: Compare your step angle and your stride and pace to that of the race walker's measurements as given on pages 363 and 364 of your textbook.

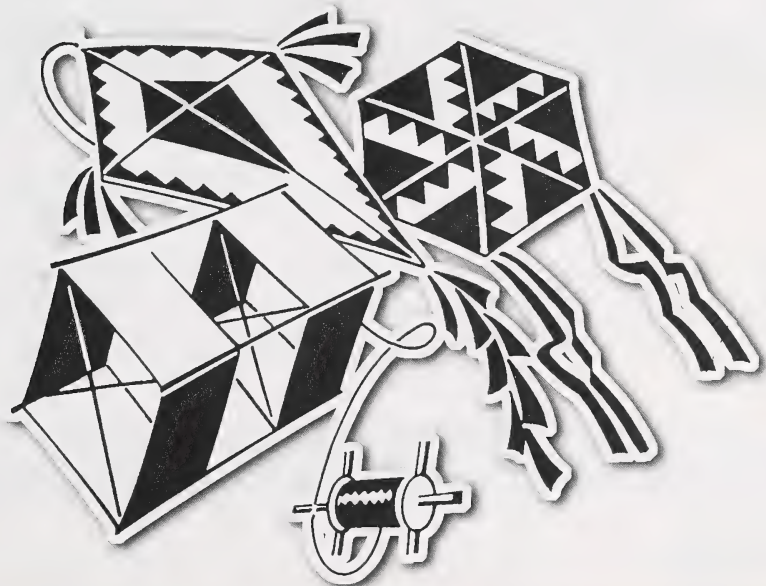
Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Enrichment, page 120.



Some questions involving trigonometry require a significant amount of thought and analysis.



2. Turn to page 380 of the textbook and answer exercise 13 of Part B of "What Should I Be Able To Do?"



Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Enrichment, pages 121–122.

MODULE PROJECT

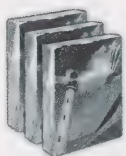
LAND SURVEYING



Completing the Project

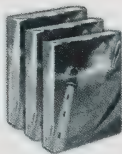
By now you should have completed your initial research for the Land Surveying project. You should have determined what information is important when surveying a property and how distances and angles are measured. You should have built your clinometer and transit and practised using these instruments.

To complete the Land Surveying project, you will be asked to use the instruments to survey an area in your schoolyard, community park, or your family's yard. The lot you survey must include a high building, tree, or other structure. You will determine the height of two of these objects for the project.

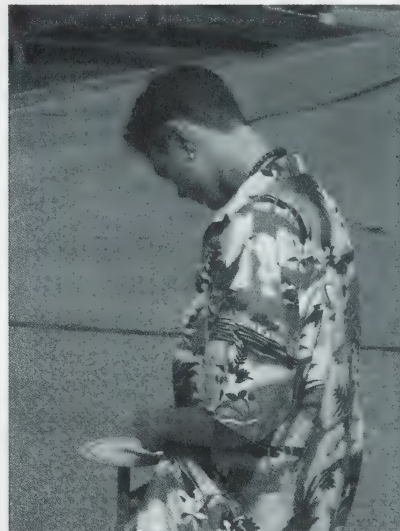


Turn to page 376 of the textbook and read the information in each of the bulleted statements. Make notes on how these points may apply to the particular land area you chose to survey.

Keep a copy of your notes in the project section of your mathematics binder. You will find these notes useful when you complete your land survey.



Before you continue your module project, turn to pages 382 and 383 of the textbook and read Part C of “What Should I Be Able To Do?” Study the information on how two particular students completed the survey project. Then, answer textbook exercises 16 and 17. This will give you more insight on how to complete your land survey.

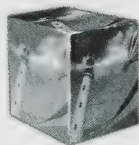


Compare your responses with the suggested answers in the Appendix, Module Project, page 123.

Module Project

Now that you have more insight into the details of completing a land survey and you are satisfied with your survey measurements, you should be ready to complete the project for Module 7. Take out the Project Booklet that accompanies this Student Module Booklet and complete the module project, Land Surveying.

Submit your completed Module 7 Project Booklet to your teacher.



Module Summary

In this module, you identified some measuring devices that are used to measure angles and elevations in surveying. You built your own measuring devices to carry out a land survey. You reviewed the basic trigonometric ratios and used these ratios to solve problems involving one or more right triangles.

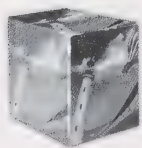
You observed the relationships that are required for the Sine Law and Cosine Law and used these laws to solve problems involving sides and angles of oblique triangles. You analysed and applied strategies for solving complex problems involving the trigonometric ratios, the Sine Law, and the Cosine Law. Finally, you completed your own land survey of a chosen area and identified the mathematics required to complete measurements in a land survey.



Knowledge of trigonometry is essential in completing land surveys as well as in determining angle and distance measurements in problems involving triangles. The single railroad that opened Canada formed the basis of a modern, complex transportation network. The tools and mathematics used by civil engineers today are much the same as in the past.

Module Assignment

To demonstrate what you have learned in this module, complete the module assignment in the Assignment Booklet.



Submit your completed Module 7 Assignment Booklet to your teacher.

COURSE SURVEY FOR APPLIED MATHEMATICS 10

(© 2000)

After you have completed the assignments in this course, please fill out this questionnaire and mail it to the address given on the last page. This course is designed in a new distance learning format, so we are interested in your responses. Your constructive comments will be greatly appreciated, as future course revisions can then incorporate any necessary improvements.

Name _____ Age ☐ under 19 ☐ 19 to 40 ☐ over 40
Address _____ File No. _____
_____ Date _____

Design

1. This course contains a series of Student Module Booklets. Do you like the idea of separate booklets?

2. Have you ever enrolled in a correspondence or distance learning course that arrived as one large volume?
☐ Yes ☐ No If yes, which style do you prefer?

3. The Student Module Booklets contain a variety of self-assessed activities. Did you find it helpful to be able to check your work and have immediate feedback?
☐ Yes ☐ No If yes, explain.

4. Were the questions and directions easy to understand?
☐ Yes ☐ No If no, explain.

5. Each module contains follow-up activities. Which type of follow-up activity did you choose?

- ☐ mainly extra help
- ☐ mainly enrichment
- ☐ a variety
- ☐ none

Did you find these activities beneficial?

- ☐ Yes ☐ No If no, explain.

6. Did you understand what was expected in the Assignment Booklets and Project Booklets?

- ☐ Yes ☐ No If no, explain.

7. The course materials were designed to be completed by students working independently at a distance. Were you always aware of what you had to do?

- ☐ Yes ☐ No If no, provide details.

8. This distance learning course may include an assortment of drawings, photographs, and charts.

a. Did you find the visuals in this course helpful?

- ☐ Yes ☐ No Comment on the lines below.

b. Did you find the variety of visuals in this course motivating?

- ☐ Yes ☐ No Comment on the lines below.

9. Suggestions for audiocassette, videocassette, and computer activities may have been included in the course. Did you complete these media activities?

☐ Yes ☐ No Comment on the lines below.

Only students enrolled in a junior high course need to complete the following question.

10. The Student Module Booklet may have directed you to work with your teacher. How well did you work as a team?

Student's comments: _____

Teacher's comments: _____

Course Content

1. Was enough detailed information provided to help you learn the expected skills and objectives?

☐ Yes ☐ No Comment on the lines below.

2. Did you find the workload reasonable?

☐ Yes ☐ No If no, explain.

3. Did you have any difficulty with the reading level?

☐ Yes ☐ No Please comment.

4. How would you assess your general reading level?

☐ poor reader ☐ average reader ☐ good reader

5. Was the material presented clearly and with sufficient depth?

☐ Yes ☐ No If no, explain.

General

1. What did you like least about the course?

2. What did you like most about the course?

Additional Comments

Only students enrolled with the Alberta Distance Learning Centre need to complete the remaining questions.

1. Did you contact the Alberta Distance Learning Centre for help or information while doing your course?

☐ Yes ☐ No If yes, approximately how many times? _____

Did you find the staff helpful?

☐ Yes ☐ No If no, explain.

2. Were you able to fax any of your assignment response pages or project response pages?

☐ Yes ☐ No If yes, comment on the value of being able to do this.

3. If you mailed your assignment response pages or project response pages, how long did it take for their return?

4. Was the feedback you received from your correspondence or distance learning teacher helpful?

☐ Yes ☐ No Please comment.

Thanks for taking the time to complete this questionnaire.
Your feedback is important to us. Please return this
questionnaire to the address on the right.

If you are enrolled at the Alberta Distance Learning Centre
and have been mailing your Assignment Booklets to ADLC,
you may return this questionnaire with the final Assignment
Booklet in the course.

Instructional Design and Development
Learning Technologies Branch
Box 4000
Barrhead, Alberta
T7N 1P4



Applied Mathematics 10

APPENDIX

GLOSSARY

Acute angle: an angle with a measure of less than 90°

Alternate angles: angles that are between two lines and on the opposite sides of a transversal that cuts the two lines

Angle of depression: the acute angle formed by the horizontal and the line of sight when the observer looks down at an object

Angle of elevation: the acute angle formed by the horizontal and the line of sight when the observer looks up at an object

Clinometer: a device used by surveyors for measuring angles of elevation and depression

Oblique triangle: a triangle that does not contain a right angle

Obtuse angle: an angle with a measure of more than 90° but less than 180°

Solving a right triangle: determining the lengths of the sides and the measures of the angles of a right triangle

Supplementary angles: two angles whose measures add up to 180°

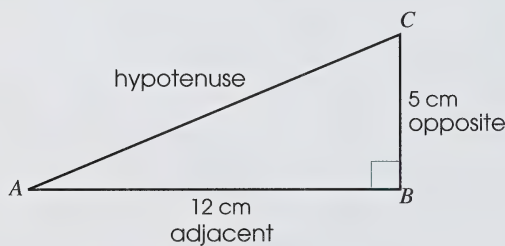
Transit: a device used by surveyors for measuring horizontal angles

SUGGESTED ANSWERS

Activity 1: Indirect Measurement

1. Textbook exercises 1 to 3 of Tutorial 7.1, "Indirect Measurement," p. 323

1. In right $\triangle ABC$, side BC (5 cm) is opposite $\angle A$, and side AB (12 cm) is the side adjacent to $\angle A$.



Determine $\tan A$.

$$\begin{aligned}\tan A &= \frac{\text{length of side opposite}}{\text{length of side adjacent}} \\ &= \frac{BC}{AB} \\ &= \frac{5}{12} \\ &\doteq 0.4167\end{aligned}$$

The ratio for $\tan A$ is about 0.4167.

Determine the measure of $\angle A$. Use the inverse function of tangent. Press $\boxed{2\text{nd}} \boxed{[\tan^{-1}]}$ on the calculator.

$$\angle A \doteq 22.6^\circ$$

The measure of $\angle A$ is about 23° .

Note: Remember to keep your calculator running between calculations.

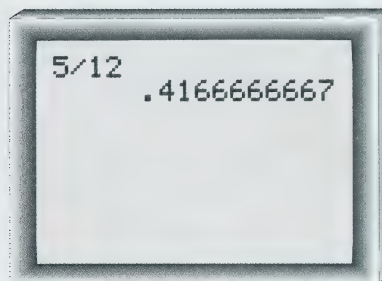
Determine the length of side AC .

You were given the length of the side opposite (5 cm), and you just determined the measure of $\angle A$ (about 23°). Side AC is the hypotenuse.

$$\begin{aligned}\sin A &= \frac{\text{length of side opposite}}{\text{length of hypotenuse}} \\ \sin A &= \frac{BC}{AC} \\ \sin 23^\circ &= \frac{5}{AC} \\ AC &= \frac{5}{\sin 23^\circ} \\ &= 13.0\end{aligned}$$

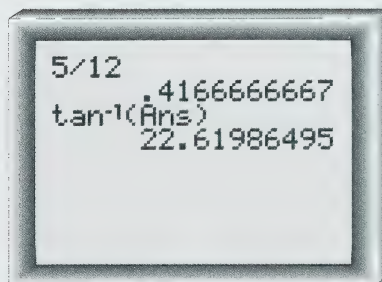
Side AC is 13 cm in length.

$\boxed{5} \boxed{+} \boxed{1} \boxed{2} \boxed{\text{ENTER}}$

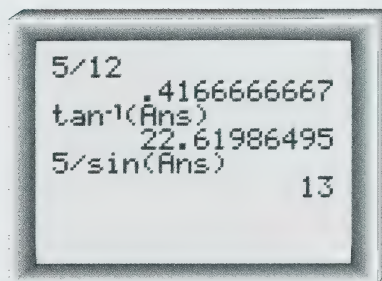


Do **not** clear the calculator.

$\boxed{2\text{nd}} \boxed{[\tan]} \boxed{2\text{nd}} \boxed{[\text{ANS}]} \boxed{)} \boxed{\text{ENTER}}$



$\boxed{5} \boxed{+} \boxed{\text{SIN}} \boxed{2\text{nd}} \boxed{[\text{ANS}]} \boxed{)} \boxed{\text{ENTER}}$



Activity 1 (continued)

Note: You can also use the Pythagorean Theorem to determine the length of side AC .

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(AC)^2 = 12^2 + 5^2$$

$$(AC)^2 = 144 + 25$$

$$(AC)^2 = 169$$

$$AC = \sqrt{169}$$

$$= 13$$

The length of side AC is 13 cm.

2. In right $\triangle PQR$, side PQ (11 cm) is opposite $\angle R$, and side PR (25 cm) is the hypotenuse.

Determine $\sin R$.

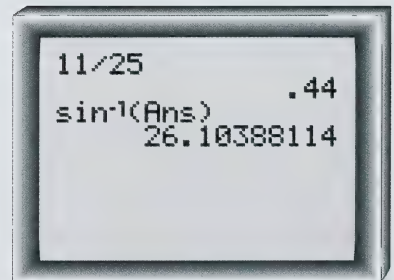
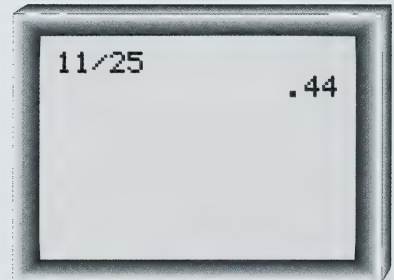
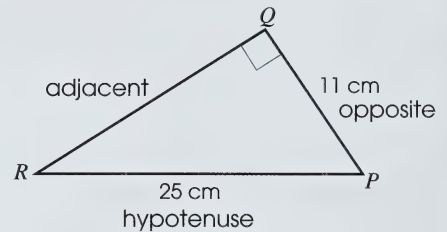
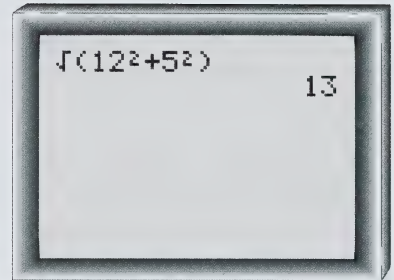
$$\begin{aligned}\sin R &= \frac{\text{length of side opposite}}{\text{length of hypotenuse}} \\ &= \frac{PQ}{PR} \\ &= \frac{11}{25} \\ &= 0.4400\end{aligned}$$

The ratio for $\sin R$ is 0.4400.

Determine the measure of $\angle R$. Use the inverse of sine.

$$\angle R \doteq 26.1^\circ$$

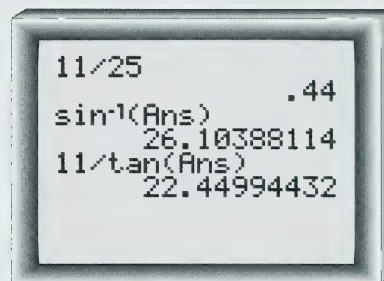
The measure of $\angle R$ is about 26° .



Determine the length of side QR .

You were given the side opposite (11 cm) $\angle R$; you just determined the measure of $\angle R$ (about 26°). Side QR is the side adjacent to $\angle R$.

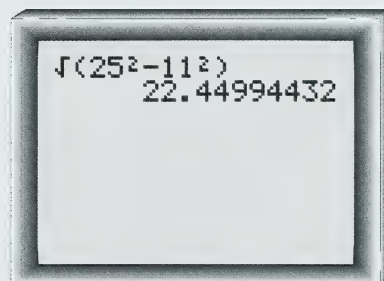
$$\begin{aligned}\tan R &= \frac{\text{length of side opposite}}{\text{length of side adjacent}} \\ \tan R &= \frac{PQ}{QR} \\ \tan 26^\circ &= \frac{11}{QR} \\ QR &= \frac{11}{\tan 26^\circ} \\ &\doteq 22.4\end{aligned}$$



Side QR is about 22 cm in length.

Note: You may also use the Pythagorean Theorem to determine the length of side QR .

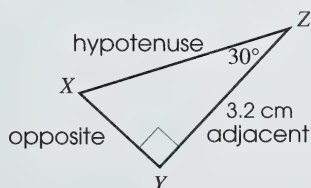
$$\begin{aligned}(RP)^2 &= (QR)^2 + (QP)^2 \\ (QR)^2 &= (RP)^2 - (QP)^2 \\ (QR)^2 &= 25^2 - 11^2 \\ (QR)^2 &= 625 - 121 \\ (QR)^2 &= 504 \\ QR &= \sqrt{504} \\ &\doteq 22.4\end{aligned}$$



The length of QR is about 22 cm .

3. In right $\triangle XYZ$, side YZ (3.2 cm) is the side adjacent to $\angle Z$, $\angle Y$ is 90° , side XZ is the hypotenuse, and side XY is the side opposite $\angle Z$.

You may determine the length of the hypotenuse by using $\cos 30^\circ$.



Activity 1 (continued)

$$\cos 30^\circ = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$$

$$\cos 30^\circ = \frac{YZ}{XZ}$$

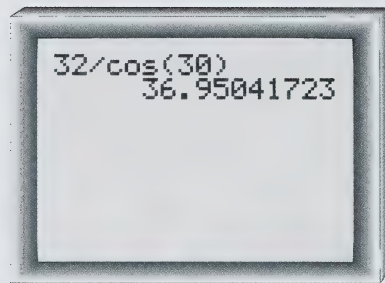
$$XZ = \frac{YZ}{\cos 30^\circ}$$

$$= \frac{32}{\cos 30^\circ}$$

$$\doteq 37.0$$

$$1 \text{ cm} = 10 \text{ mm}$$

$$3.2 \text{ cm} = 32 \text{ mm}$$



The length of XZ is about 37 mm.

You may determine the length of the opposite side (XY) by finding $\tan 30^\circ$.

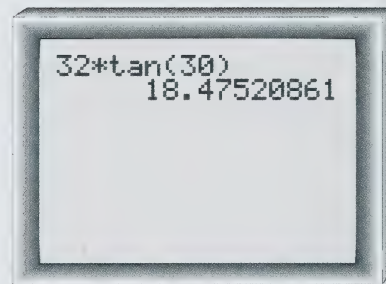
$$\tan 30^\circ = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$\tan 30^\circ = \frac{XY}{YZ}$$

$$XY = YZ \times \tan 30^\circ$$

$$= 32 \times \tan 30^\circ$$

$$\doteq 18.5$$



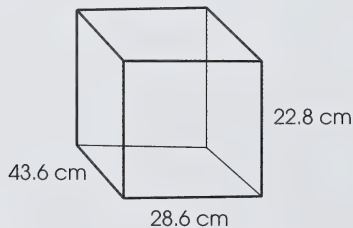
The length of XY is about 18 mm.

Note: You could have solved this triangle by first finding the measure of $\angle YXZ$ and then using the trigonometric ratios. You could also have used the Pythagorean Theorem to find the length of the third side.

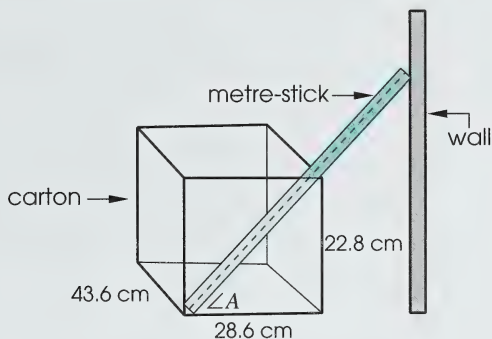
2. Textbook exercises 1 to 8 of “Investigation 1: Indirect Measurement,” p. 324

Answers will vary. One example follows.

- The dimensions of the sample box are $43.6 \text{ cm} \times 28.6 \text{ cm} \times 22.8 \text{ cm}$.



2. The diagram shows the angle of inclination, $\angle A$, of the metre-stick. The metre-stick makes a small right triangle with the sides of the box.



The side opposite $\angle A$ is 22.8 cm;
the side adjacent to $\angle A$ is 28.6 cm.

Note: The diagram is **not** drawn to scale.

Find the tangent ratio.

$$\begin{aligned}\tan A &= \frac{\text{length of opposite side}}{\text{length of adjacent side}} \\ &= \frac{22.8}{28.6} \\ &\doteq 0.7972\end{aligned}$$

Therefore, $\tan A$ is about 0.7972.

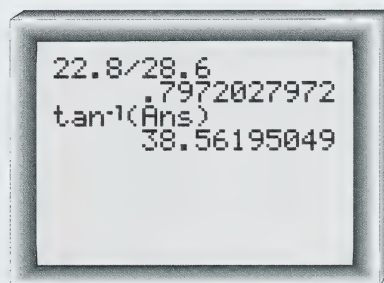
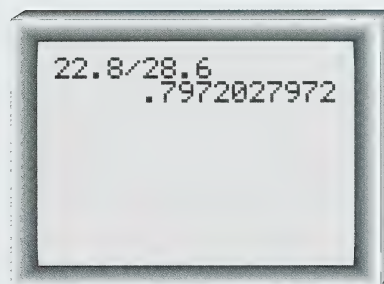
Use the inverse function of tangent to find the measure of $\angle A$.

$$\angle A \doteq 38.6^\circ$$

Therefore, $\angle A$ is about 39° .

The angle of inclination of the metre-stick is about 39° .

Do **not** clear your calculator. You will need your unrounded answer in the next part of the investigation.



3. The diagram in exercise 2 shows the angle of inclination of the metre-stick, $\angle A$. The metre-stick makes a large right triangle with the wall and the floor. In this large right triangle, the length of the metre-stick (100 cm) is the hypotenuse, and the height the metre-stick reaches up the wall (h) is the side opposite $\angle A$.

Activity 1 (continued)

Use the sine ratio to calculate the height the metre-stick reaches up the wall.

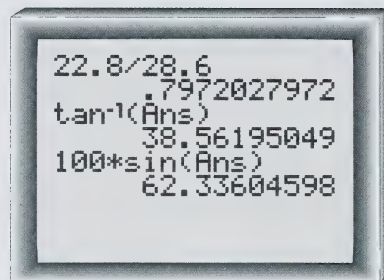
$$\sin A = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

$$\sin 39^\circ \doteq \frac{h}{100}$$

$$h \doteq 100 \times \sin 39^\circ$$

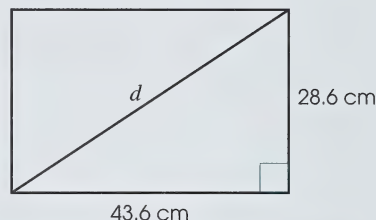
$$\doteq 62.34$$

The calculated height that the metre-stick reaches up the wall is about 62.3 cm.



- Answers will vary. Based on this example, the measured height that the metre-stick reached up the wall is 62.8 cm. The calculated height and the measured height are close. You would probably assume that the calculated answer is more accurate.
- You need to find the length of the diagonal of the bottom of the box and the measured distance on the metre-stick from the lower corner of the box to the upper diagonal corner of the box.

You can use the Pythagorean Theorem to calculate the length of the diagonal of the bottom of the box.



$$d^2 = \ell^2 + w^2$$

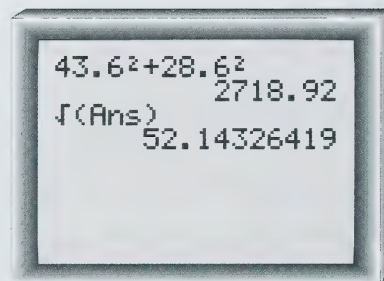
$$d^2 = 43.6^2 + 28.6^2$$

$$d^2 = 2718.92$$

$$d = \sqrt{2718.92}$$

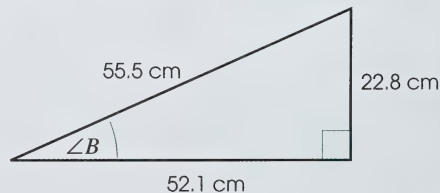
$$\doteq 52.14$$

The length of the diagonal of the base of the box is about 52.1 cm.



The measured length of the diagonal from the lower corner to the upper corner is 55.5 cm.

The angle of inclination can be determined using the calculated diagonal and the tangent ratio or the measured length of the diagonal from corner to corner and the sine ratio.



Method 1: Using the Tangent Ratio

$$\begin{aligned}\tan B &= \frac{\text{length of opposite side}}{\text{length of adjacent side}} \\ &= \frac{22.8}{52.1} \\ &\doteq 0.4373\end{aligned}$$

Therefore, $\tan B$ is about 0.4373.

Use the inverse of tangent to find the measure of $\angle B$.

$$\angle B \doteq 23.6^\circ$$

The angle of inclination is about 24° .

Method 2: Using the Sine Ratio

$$\begin{aligned}\sin B &= \frac{\text{length of opposite side}}{\text{length of hypotenuse}} \\ &= \frac{22.8}{55.5} \\ &\doteq 0.4108\end{aligned}$$

Therefore, $\sin B$ is about 0.4108.

Use the inverse of sine to find the measure of $\angle B$.

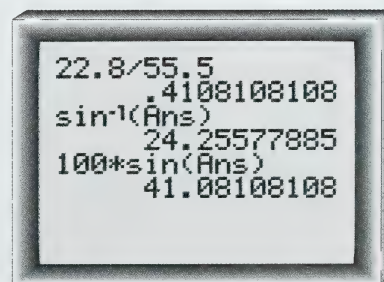
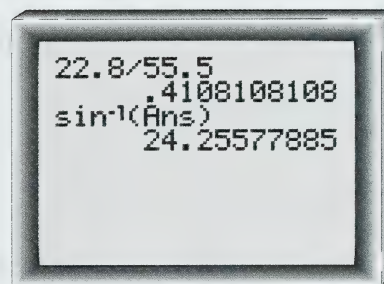
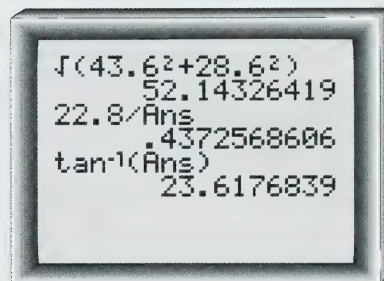
$$\angle B \doteq 24.3^\circ$$

The angle of inclination is about 24° .

6. The length of the metre-stick (100 cm) is the hypotenuse, and the height up the wall (h) is the side opposite $\angle B$ (about 24°). Therefore, use the sine ratio to calculate the height.

$$\begin{aligned}\sin B &= \frac{\text{length of opposite side}}{\text{length of hypotenuse}} \\ \sin 24^\circ &= \frac{h}{100} \\ h &= 100 \times \sin 24^\circ \\ &\doteq 41.1\end{aligned}$$

The height of the metre-stick is about 41 cm.



Activity 1 (continued)

- Answers may vary. The measured height of the metre-stick is 41 cm. The measured result and the rounded, calculated result are the same (when rounded to the nearest centimetre).
- The angle of inclination is less in textbook exercise 5 because the length of the adjacent side in exercise 5 is longer.

$$28.6 \text{ cm} < 43.6 \text{ cm}$$

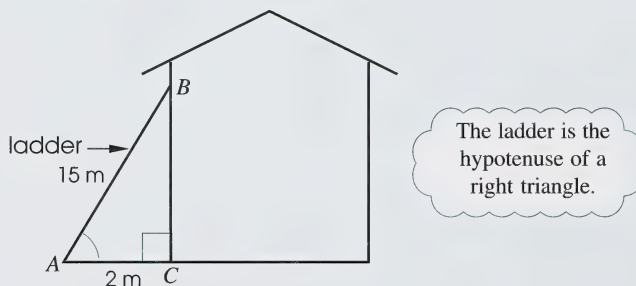
$$\angle A \doteq 39^\circ > \angle B \doteq 24^\circ$$

3. Textbook exercises 1 and 2 of “Discussing the Ideas,” p. 326

- Diagrams allow you to visualize the angles and distances involved. They create a picture of the problem.
- Yes, the calculations will be affected by the accuracy of the measurements. The precision of the measuring instrument will also affect your calculations. A calculation using a measurement to the nearest centimetre will be different from one that uses a measurement to the nearest tenth of a centimetre.

4. Textbook exercises 1, 2, 5, and 6 of “Exercises: Checking Your Skills,” pp. 326 and 327

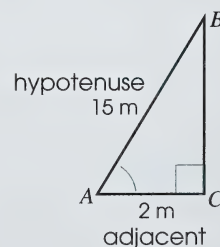
- Step 1:** Draw a diagram to illustrate the problem.



Step 2: Clarify the problem. Draw a diagram of the right triangle.

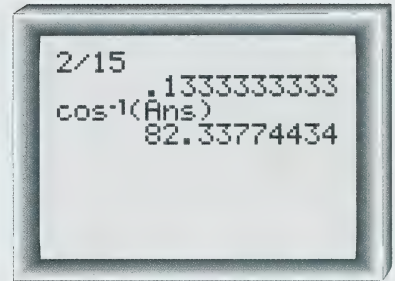
The angle between the ladder and the ground is $\angle A$.

The length of the hypotenuse is 15 m and the length of the adjacent side is 2 m.



Step 3: Use cosine to calculate the measure of $\angle A$.

$$\begin{aligned}\cos A &= \frac{\text{length of adjacent side}}{\text{length of hypotenuse}} \\ &= \frac{AC}{AB} \\ &= \frac{2}{15} \\ &\doteq 0.1333\end{aligned}$$

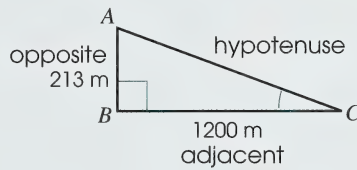
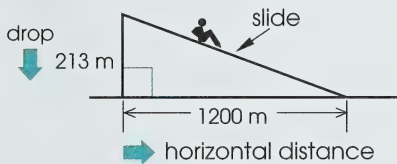


Find the measure of $\angle A$ by using the inverse of cosine.

$$\angle A \doteq 82.3^\circ$$

The angle the ladder makes with the ground is about 82° .

2. Draw a diagram to illustrate the situation, and then clarify the problem.

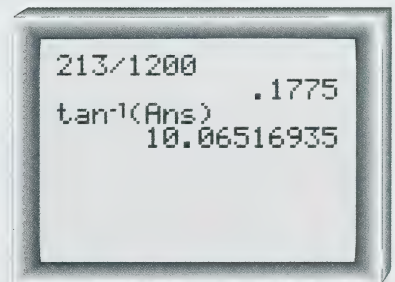


The slide is the hypotenuse of a right triangle. The measure of $\angle C$ is the angle of inclination of the slide. The length of the drop (213 m) is the side opposite and the length of the horizontal distance covered (1200 m) is the side adjacent.

- a. You may use the tangent ratio or the Pythagorean Theorem to find the length of the slide.

Method 1: Using the Tangent Ratio

$$\begin{aligned}\tan C &= \frac{\text{length of side opposite}}{\text{length of side adjacent}} \\ &= \frac{AB}{BC} \\ &= \frac{213}{1200} \\ &= 0.1775\end{aligned}$$



Find the measure of $\angle C$. Use the inverse of tangent.

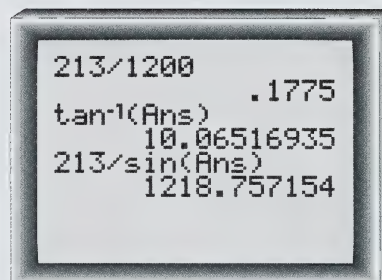
$$\angle C \doteq 10.1^\circ$$

Activity 1 (continued)

Find the length of the hypotenuse by using the sine ratio.

$$\sin C = \frac{\text{length of side opposite}}{\text{length of hypotenuse}}$$

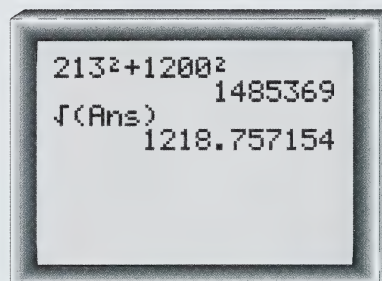
$$\begin{aligned} AC &= \frac{AB}{\sin C} \\ &= \frac{213}{\sin 10^\circ} \\ &\doteq 1218.8 \end{aligned}$$



The length of the slide is about 1219 m.

Method 2: Using the Pythagorean Theorem

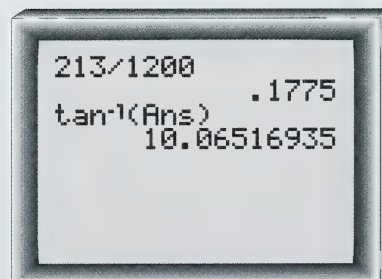
$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 213^2 + 1200^2 \\ c^2 &= 148\,536\,9 \\ c &= \sqrt{148\,536\,9} \\ c &\doteq 1218.8 \end{aligned}$$



The length of the slide is about 1219 m.

- b. The measure of $\angle C$ is the angle of inclination. **Note:** If you used the trigonometric ratio in part a. (Method 1), you have already determined the measure of $\angle C$. If not, use the tangent ratio.

$$\begin{aligned} \tan C &= \frac{\text{length of opposite side}}{\text{length of adjacent side}} \\ &= \frac{AB}{BC} \\ &= \frac{213}{1200} \\ &= 0.1775 \end{aligned}$$

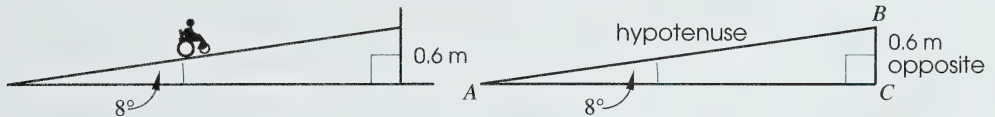


Find the measure of $\angle C$. Use the inverse of tangent.

$$\angle C \doteq 10.1^\circ$$

The angle of inclination of the slide is about 10° .

5. Draw a diagram to illustrate the situation, and then clarify the problem.



The ramp is the hypotenuse of a right triangle. The measure of $\angle A (8^\circ)$ is the angle of inclination. The length of the rise is the side opposite $\angle A$. Use the sine ratio to find the length of the ramp (the hypotenuse).

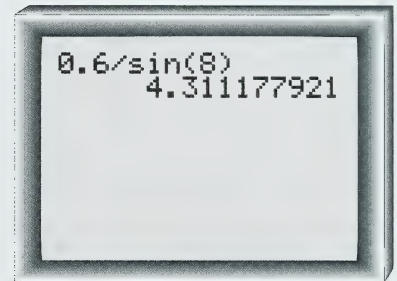
$$\sin A = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

$$\sin A = \frac{BC}{AB}$$

$$\sin 8^\circ = \frac{0.6}{AB}$$

$$AB = \frac{0.6}{\sin 8^\circ}$$

$$\doteq 4.31$$



The length of the ramp is about 4.3 m.

6. The diagram illustrating the problem is drawn in the textbook. The angle of depression is equal to the angle of inclination (see the definitions on page 417 of the textbook). The height of the plane (6000 m) is the length of the side opposite $\angle A$. The distance the plane has to travel is equal to the side adjacent. Use the tangent ratio to calculate the length of the side adjacent.

$$\tan A = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

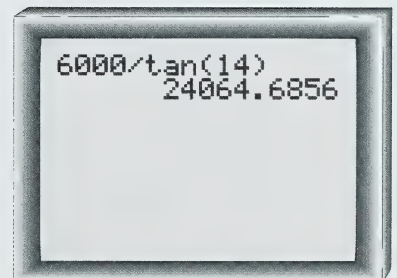
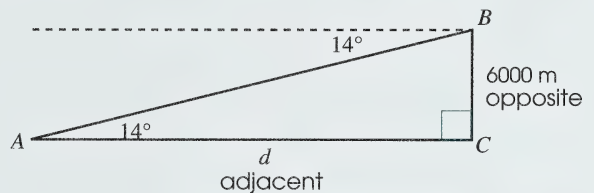
$$\tan A = \frac{BC}{AC}$$

$$\tan 14^\circ = \frac{6000}{AC}$$

$$AC = \frac{6000}{\tan 14^\circ}$$

$$\doteq 24\,064.7$$

$$1\text{ km} = 1000\text{ m}$$



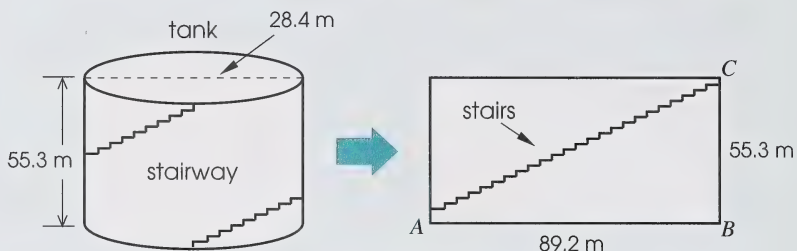
The airplane has to fly about 24 065 m (or 24 km) to reach the shore.

Activity 1 (continued)

5. Textbook exercise 7 of “Exercises: Extending Your Thinking,” p. 327

7. Draw and label a diagram and clarify the problem. If the tank is cut open and spread out, the cylinder becomes a rectangle. To find the length of the rectangle, you need to find the circumference of the cylinder.

$$\begin{aligned} C &= \pi d \\ &= \pi \times 28.4 \\ &\doteq 89.22 \end{aligned}$$



Note: The diagrams are **not** drawn to scale.

The stairs are the hypotenuse of a right-angle triangle. The height of the tank (55.3 m) is the side opposite. The circumference of the tank (about 89.2 m) is the side adjacent. Use the tangent ratio to find the angle of inclination ($\angle A$) of the stairway.

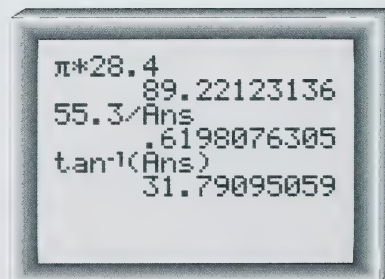
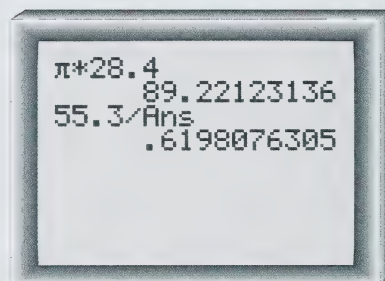
$$\begin{aligned} \tan A &= \frac{\text{length of opposite side}}{\text{length of adjacent side}} \\ &= \frac{BC}{AB} \\ &= \frac{55.3}{89.2} \\ &\doteq 0.6198 \end{aligned}$$

Find the measure of $\angle A$.

Use the inverse function of tangent.

$$\angle A \doteq 31.79^\circ$$

The angle of inclination of the stairway is about 31.8° .



6. Textbook exercise “Communicating the Ideas,” p. 327

Answers will vary. The following are suggested methods. You may have used some other method to determine the height of the flagpole.

Direct Measurement

One way to directly measure the height of a flagpole is to tie a long tape measure to the rope (above the position of the flag) and raise the flag and tape measure up to the top of the pole. Record the measure of the pole at ground level.

Advantages

- This method is quick and easy to do.
- No special equipment, such as a cherry picker (a special crane), is necessary.

Disadvantages

- The accuracy of the measurement may be to the nearest 0.5 m since the tape may not quite reach the top of the pole.

Indirect Measurement

Measure the distance from a convenient marked point on the ground to the base of the flagpole. Use an angle-measuring device (a clinometer) to measure the angle of elevation from the marked point to the top of the flagpole. Use an appropriate trigonometric relation to calculate the height of the flagpole using the measured angle and the distance to the base of the flagpole.

Advantages

- The measurement can be done without a long measuring tape (other methods, such as pacing, could be used to determine the distance to the base of the flagpole).
- The measurement may be more accurate than a direct measurement (unless a method such as pacing was used).

Disadvantages

- Equipment such as an angle-measuring device and a calculator are required.
- The accuracy of the measurement depends on the equipment used.

Activity 2: Solving Problems Using More Than One Right Triangle

1. Yes, the two triangles are in the same plane if you assume that both trees are vertical and the ground is level.
2. No, point C does not have to be halfway between the two trees. It is given as halfway for simplicity.

Activity 2 (continued)

3. The tangent function is used to determine the heights of the trees because the tangent function uses the opposite side and the adjacent side. In the diagram, the known sides of the triangles are the opposite sides and the adjacent sides.
4. Yes, the two triangles in the diagram on page 334 are in the same plane.
5. The lower triangle is used to determine the length of x because the length of the opposite side is known in the lower triangle: it is 25 m.

Note the following about Appendix answers:

In the given solutions, answers are given in rounded form at each step. However, in working with a calculator in multi-step problems, the calculator is kept running and unrounded answers are used in subsequent steps. Using unrounded answers allows greater precision.

Note, also, that diagrams are not drawn to scale.

6. Textbook exercises 1, 2, 3, 5, 7, and 8 of “Exercises: Checking Your Skills,” pp. 337 to 339

1. Make a plan. To find the length of BC , find the length of BD and the length of CD , and then add the two lengths together.

Step 1: Find the length of BD .

Use a detailed diagram to help identify the trigonometric function.

Side BD is the side opposite $\angle 30^\circ$; side AD is the side adjacent to $\angle 30^\circ$. Therefore, use tangent.

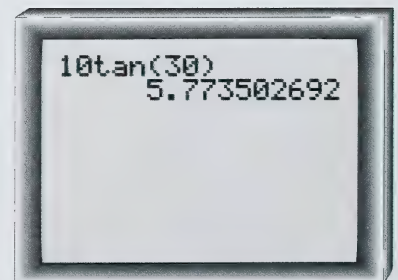
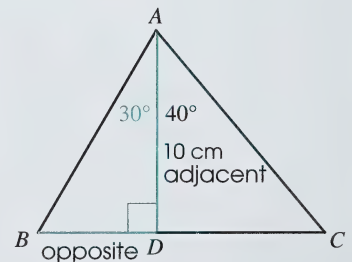
$$\tan 30^\circ = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$\tan 30^\circ = \frac{BD}{AD}$$

$$\tan 30^\circ = \frac{BD}{10}$$

$$BD = 10 \tan 30^\circ$$

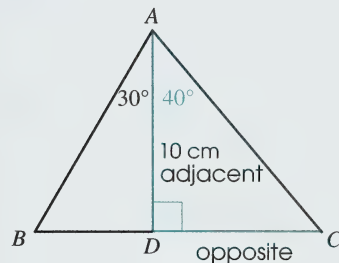
$$\doteq 5.8$$



Step 2: Find the length of CD .

Use a detailed diagram to identify the trigonometric function.

Side CD is the side opposite $\angle 40^\circ$; side AD is the side adjacent to $\angle 40^\circ$. Therefore, use tangent.



$$\tan 40^\circ = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$\tan 40^\circ = \frac{CD}{AD}$$

$$\tan 40^\circ = \frac{CD}{10}$$

$$CD = 10 \tan 40^\circ$$

$$\doteq 8.4$$

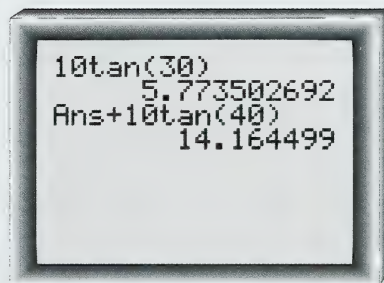
Step 3: Find the length of BC .

$$\text{length of } BC = \text{length of } BD + \text{length of } CD$$

$$\doteq 5.8 + 10 \tan 40^\circ$$

$$\doteq 14.2$$

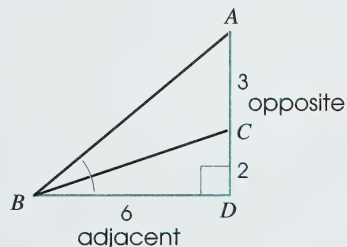
Rounded to the nearest centimetre, the length of BC is about 14 cm.



2. Make a plan. Notice that $\triangle ABD$ is not a right triangle. To find the measure of $\angle ABC$, you need to find the measure of $\angle ABD$ and $\angle CBD$, and then subtract the measure of $\angle CBD$ from the measure of $\angle ABD$.

Step 1: Find the measure of $\angle ABD$.

$$\begin{aligned} \text{length of side } AD &= \text{length of } AC + \text{length of } CD \\ &= 3 + 2 \\ &= 5 \end{aligned}$$

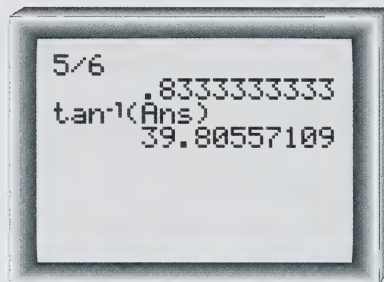
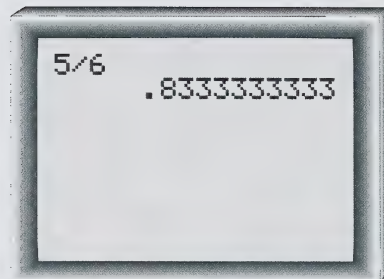


Activity 2 (continued)

$$\begin{aligned}\tan \angle ABD &= \frac{\text{length of opposite side}}{\text{length of adjacent side}} \\ &= \frac{AD}{BD} \\ &= \frac{5}{6} \\ &\doteq 0.8333\end{aligned}$$

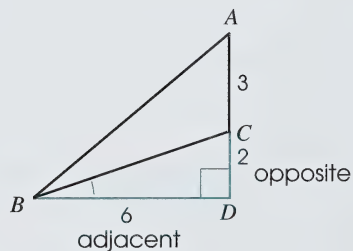
Use the inverse function of tangent to find the measure of $\angle ABD$.

$$\angle ABD \doteq 39.8^\circ$$



Step 2: Find the measure of $\angle CBD$.

$$\begin{aligned}\tan \angle CBD &= \frac{\text{length of opposite side}}{\text{length of adjacent side}} \\ &= \frac{CD}{BD} \\ &= \frac{2}{6} \\ &\doteq 0.3333\end{aligned}$$



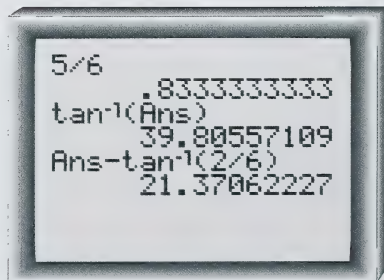
Use the inverse function of tangent to find the measure of $\angle CBD$.

$$\angle CBD \doteq 18.4^\circ$$

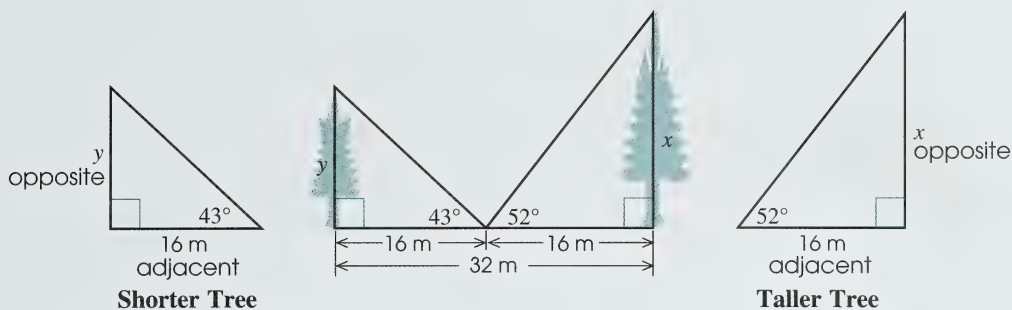
Step 3: Find the measure of $\angle ABC$. Subtract the measure of $\angle CBD$ from the measure of $\angle ABD$.

$$\begin{aligned}\angle ABC &= \angle ABD - \angle CBD \\ &\doteq 39.8^\circ - 18.4^\circ \\ &\doteq 21.4^\circ\end{aligned}$$

Therefore, the measure of $\angle ABC$ is about 21° .



3. a. Draw and label a diagram to illustrate the problem.

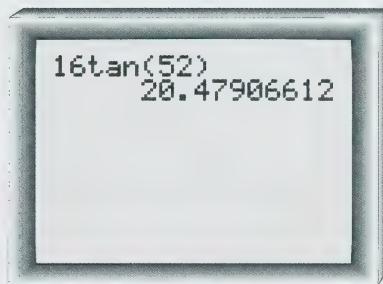


- b. **Step 1:** Make a plan. The height of each tree is the side opposite the angle of inclination. Half the distance between the trees is the side adjacent to each angle of inclination. Use the tangent ratio to find the height of each tree.

Step 2: Find the height of the taller tree.

$$\begin{aligned}\tan 52^\circ &= \frac{\text{length of opposite side}}{\text{length of adjacent side}} \\ &= \frac{x}{16} \\ x &= 16 \tan 52^\circ \\ &\approx 20.5\end{aligned}$$

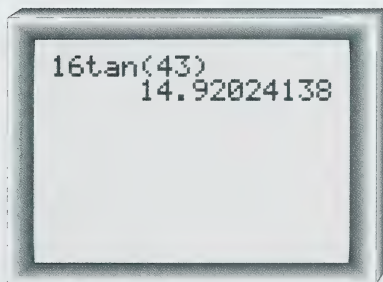
The height of the taller tree is about 20.5 m.



Step 3: Find the height of the shorter tree.

$$\begin{aligned}\tan 43^\circ &= \frac{\text{length of opposite side}}{\text{length of adjacent side}} \\ &= \frac{y}{16} \\ y &= 16 \tan 43^\circ \\ &\approx 14.9\end{aligned}$$

The height of the shorter tree is about 14.9 m.



Activity 2 (continued)

- c. **Step 1:** Make a plan. Find the difference in the heights of the trees. Then, express the difference as a percentage of the height of the shorter tree.

Step 2: Find the difference in the heights of the trees.

difference in height = height of taller tree – height of shorter tree

$$\doteq 20.5 - 14.9$$

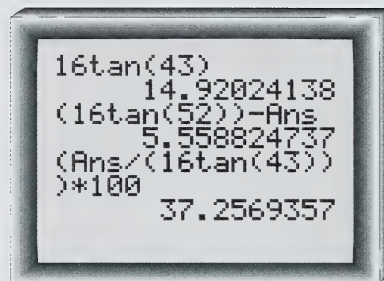
$$\doteq 5.6$$

Step 3: Find the difference in height as a percentage of the height of the shorter tree.

$$\text{difference as a \%} = \frac{\text{difference in heights}}{\text{height of shorter tree}} \times 100$$

$$\doteq \frac{5.6}{14.9} \times 100$$

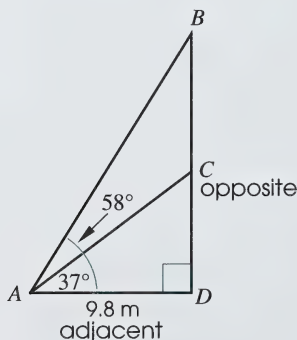
$$\doteq 37.3$$



The difference in the heights of the trees as a percentage of the height of the shorter tree is about 37%.

5. Make a plan. Find the height of the tower; then find the height of the anchor point from the ground. To find how far the top of the tower is from the anchor point, subtract the height of the anchor point from the height of the tower.

Draw and label a diagram to illustrate the situation. Then, clarify the problem.



Step 1: Find the height of the tower. The height of the tower is the length of BD .

Side BD is the side opposite $\angle BAD$. Side AD (9.8 m) is the side adjacent to $\angle BAD$. Use the tangent ratio to find the length of BD .

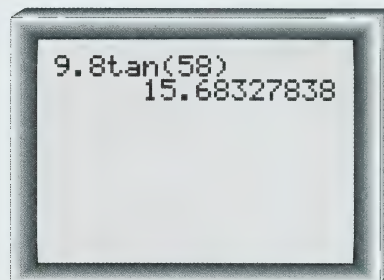
$$\tan BAD = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$\tan BAD = \frac{BD}{AD}$$

$$\tan 58^\circ = \frac{BD}{9.8}$$

$$BD = 9.8 \tan 58^\circ$$

$$= 15.68$$



The height of the tower is about 15.7 m.

Step 2: Find the distance from the guy wire to the top of the tower.

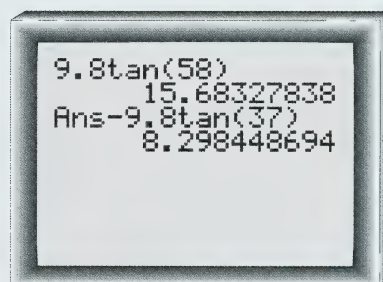
First, find the height of the point of attachment of the guy wire from the ground. The height of the attachment is the length of CD . Side CD is the side opposite $\angle CAD$. Side AD (9.8 m) is the side adjacent to $\angle CAD$. Use the tangent ratio to find the length of CD .

$$\tan CAD = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$\tan 37^\circ = \frac{CD}{9.8}$$

$$CD = 9.8 \tan 37^\circ$$

$$= 7.38$$



Now, find the distance from the guy wire to the top of the tower.

Subtract the height of the anchor point from the height of the tower.

$$\text{distance} = \text{height of tower} - \text{height of wire to the ground}$$

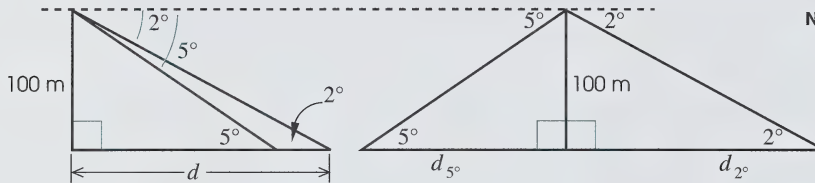
$$= 15.68 - 7.38$$

$$= 8.30$$

The distance from the point of attachment of the guy wire on the tower to the top of the tower is about 8.3 m.

Activity 2 (continued)

7. Clarify the problem. Draw two diagrams showing the angles and distances of the fires from the tower. The diagram on the left represents the fires being on the same side of the tower; the diagram on the right represents the fires being on opposite sides of the tower. Note that each angle of inclination is the same as its corresponding angle of depression: they are alternate angles.



Note: The diagrams are not drawn to scale.

- a. Make a plan. The distance of each fire from the tower is represented by the side adjacent to the angle. The height of the tower is the side opposite the angle. Use the tangent ratio to calculate the distance to each fire. To find the distance between the fires on the same side of the tower, subtract the shorter distance from the longer distance.

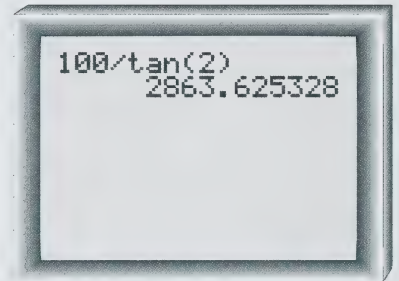
Step 1: Find the distance of the fire at the angle of 2° .

$$\tan 2^\circ = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$\tan 2^\circ = \frac{100}{d}$$

$$d = \frac{100}{\tan 2^\circ}$$

$$\doteq 2863.6$$

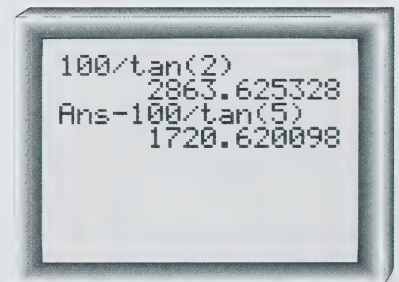


Step 2: Find the distance of the fire at the angle of 5° .

$$\tan 5^\circ = \frac{100}{d}$$

$$d = \frac{100}{\tan 5^\circ}$$

$$\doteq 1143.0$$



Step 3: To determine the distance between the fires on the same side of the tower, subtract the shorter distance from the longer distance.

$$\begin{aligned} \text{distance between fires} &= \text{distance of farther fire} - \text{distance of closer fire} \\ &\doteq 2863.6 - 1143.0 \\ &\doteq 1720.6 \end{aligned}$$

When the fires are on the same side of the tower, the distance between the fires is about 1721 m.

- b. The distance between the fires when the fires are on the opposite sides of the tower is equal to the sum of the distance each fire is from the tower.

distance between fires = distance of farther fire + distance of closer fire

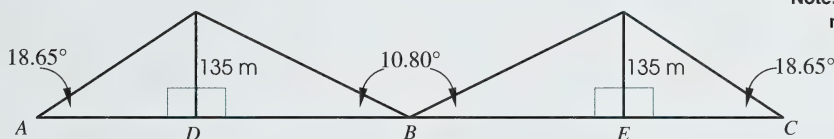
$$\doteq 2863.6 + 1143.0$$

$$\doteq 4006.6$$

When the fires are on opposite sides of the tower, the distance between the fires is about 4007 m.

Use unrounded values on your calculator.

8. Draw and label a diagram to illustrate the problem.



Note: The diagrams are **not** drawn to scale.

Make a plan. The side opposite each angle of elevation is known. Find the side adjacent to each angle of elevation. Use the tangent ratio to determine the length of each adjacent side.

Step 1: Find the length of AD .

$$\tan A = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$\tan 18.65^\circ = \frac{135}{AD}$$

$$AD = \frac{135}{\tan 18.65^\circ}$$

$$\doteq 400.0$$

Step 2: Find the length of BD .

$$\tan B = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$\tan 10.80^\circ = \frac{135}{BD}$$

$$BD = \frac{135}{\tan 10.80^\circ}$$

$$\doteq 707.7$$

Activity 2 (continued)

Step 3: Find the lengths of BE and CE .

Since the bridge is symmetrical, $AD = CE$ and $BD = BE$.

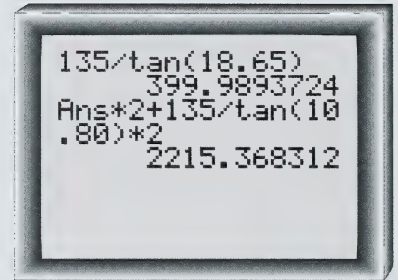
$$\begin{aligned}AD &\doteq 400.0 & BD &\doteq 707.7 \\ \therefore CE &\doteq 400.0 & \therefore BE &\doteq 707.7\end{aligned}$$

Step 4: Find the total length by adding the segment lengths.

$$\begin{aligned}\text{Total length} &= AD + BD + BE + CE \\ &\doteq 400.0 + 707.7 + 400.0 + 707.7 \\ &\doteq 2215.4\end{aligned}$$

The total length of the bridge is about 2215 m.

Note: Do **not** clear the calculator between steps.
Answers are given as unrounded figures.

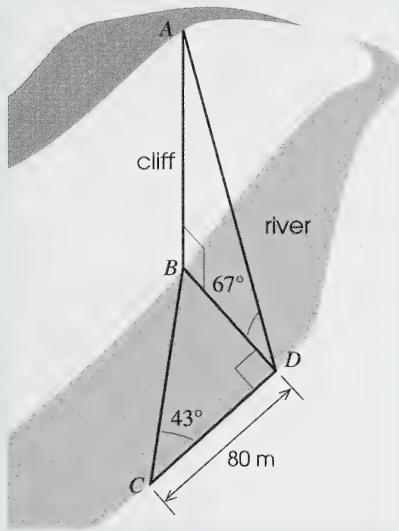


7. a. The triangles need to be at right angles to each other since $\triangle SRB$ is using a horizontal measurement and $\triangle TBR$ requires a vertical measurement.
- b. The common side to $\triangle TBR$ and $\triangle SRB$ is BR .
8. Textbook exercises 1, 2, and 3 of “Discussing the Ideas,” p. 336
1. The right angles will give an indication as to the position or orientation of the right triangles.
 2. You should try to think of a plan for solving a problem at the start of the problem-solving process.
 3. It was important to draw the triangles because the diagrams showed which sides were opposite, adjacent, and the hypotenuse in each triangle.

9. Textbook exercise 9 of “Exercises: Checking Your Skills,” p. 339

9. **Note:** Diagrams are **not** drawn to scale.

Step 1: Draw and label a diagram. Make a plan for solving the problem.



Triangle BCD is in a horizontal plane and $\triangle ABD$ is in a vertical plane.

Side BD is common to both triangles. This length is the distance across the river.

The height of the cliff is AB . To determine the height, you need to know the length of one side of $\triangle ABD$. Use the common side, BD .

Step 2: Draw $\triangle BCD$. Label the sides in terms of $\angle C$.

Use the tangent ratio to find the length of side BD , which is the distance across the river.

$$\tan C = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

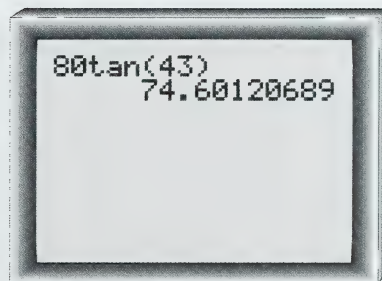
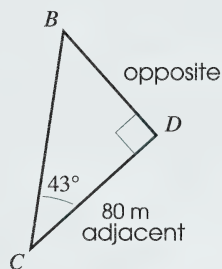
$$\tan 43^\circ = \frac{BD}{80}$$

$$BD = 80 \tan 43^\circ$$

$$\doteq 74.6$$

Side BD is about 75 m.

Do **not** clear the calculator.



Activity 2 (continued)

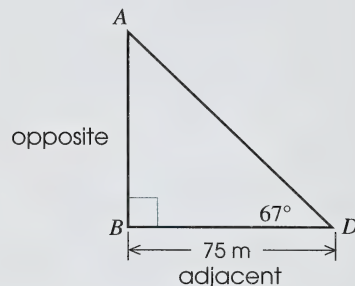
Step 3: Solve for AB , the height of the cliff.

Draw and label $\triangle ABD$.

Side AB is the side opposite $\angle 67^\circ$. Side BD is the side adjacent $\angle 67^\circ$.

AB = height of cliff

BD = width of river



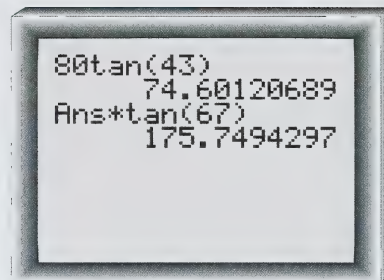
Use the tangent ratio to find the length of AB .

$$\tan D = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$\tan D = \frac{AB}{BD}$$

$$\tan 67^\circ \doteq \frac{AB}{74.6}$$

$$AB \doteq 74.6 \tan 67^\circ \\ \doteq 175.7$$



The height of the cliff is about 176 m.

Remember to use the unrounded answers in the calculations.

10. Textbook question “Communicating the Ideas,” p. 340

The steps you list should be similar to the following:

Step 1: Draw a diagram illustrating the situation. Label the diagram as completely as possible.

Step 2: Write a plan of what you need to do to solve the problem.

Step 3: Select the appropriate trigonometric ratio required to solve each part of the problem.

Step 4: Follow your plan and complete the solution of the problem.

Activity 3: Sine and Cosine for Angles from 0° to 180°

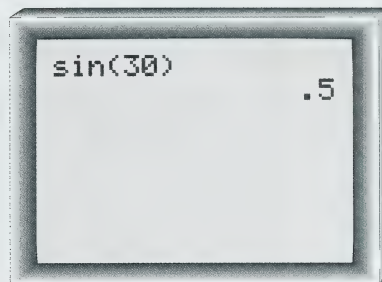
1. Textbook exercises 1 to 8 of “Investigation 1: Extend the Concept of Sine for Angles between 0° and 180° ,” pp. 341 and 342

1. The calculator display indicates the sine of 30° is 0.5.

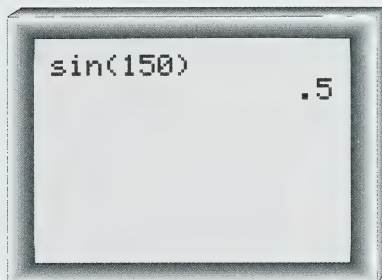
Press **SIN** **3** **0** **)** **ENTER**.

↑
closing parenthesis

Remember: Get into the habit of using the closing parenthesis.



2. The calculator display indicates the sine of 150° is 0.5.



3. To change the WINDOW setting, press **WINDOW** and use the arrow keys to move the cursor to the required position. Use the number keys to enter the new numbers. Refer to your calculator manual if you are having difficulty changing the WINDOW settings. Press **2nd** **[QUIT]** to go back to the main screen.
4. The graph displayed on your calculator screen should look like the one in textbook exercise 4 on page 342.
5. You should have made a table similar to the one in textbook exercise 5 on page 342.

Activity 3 (continued)

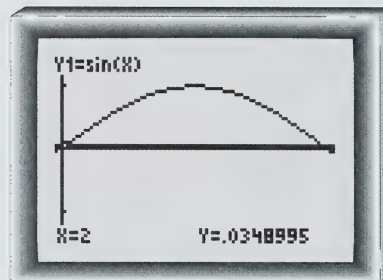
6. Follow these steps to complete the chart. This example uses the first value given in the chart. Answers are rounded to four decimal places.

Step 1: Graph $y = \sin x$.

Step 2: Use the **TRACE** feature and arrow keys to find the y -value when $x = 2$.

$$y = \sin x$$

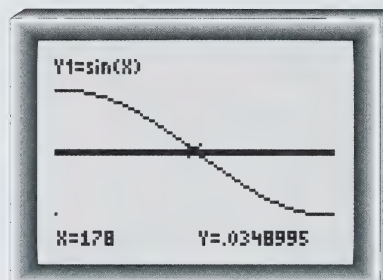
$$0.0349 \doteq \sin 2^\circ$$



Step 3: Use the **TRACE** feature and arrow keys to find any other x -value when $y \doteq 0.0349$.

$$y = \sin x$$

$$0.0349 \doteq \sin 178^\circ$$



Your completed table should look like the following.

angle x	$y = \sin x$	Another value for x with the same value for y
2°	0.0349	178°
10°	0.1736	170°
28°	0.4695	152°
32°	0.5299	148°
46°	0.7193	134°
58°	0.8480	122°
62°	0.8829	118°
80°	0.9848	100°
88°	0.9994	92°

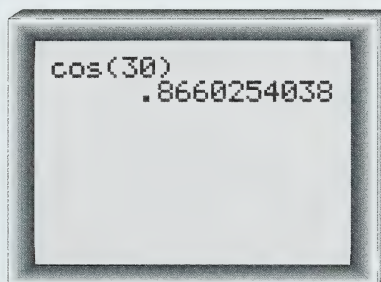
7. The sine of the angle in the first column is the same as the sine of the angle in the last column; for example, $\sin 2^\circ = \sin 178^\circ$. Also, the sum of the two angles is 180° .
8. The completed equations should look like the following:

$\sin 2^\circ = \sin 178^\circ$	$\sin 146^\circ = \sin 34^\circ$
$\sin 10^\circ = \sin 170^\circ$	$\sin 154^\circ = \sin 26^\circ$
$\sin 38^\circ = \sin 142^\circ$	$\sin 160^\circ = \sin 20^\circ$
$\sin 66^\circ = \sin 114^\circ$	$\sin 178^\circ = \sin 2^\circ$

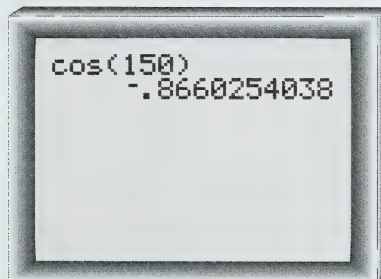
2. Textbook exercise 1 of “Investigation 2: Extend the Concept of Cosine for Angles between 0° and 180° ,” p. 342

1. These exercises are based on “Investigation 1,” replacing *sine* with *cosine*.

1. The calculator display indicates the cosine of 30° is about 0.8660.



2. The calculator display indicates the cosine of 150° is about -0.8660 . Notice the number value is the same as for cosine 30° , but the sign is negative.

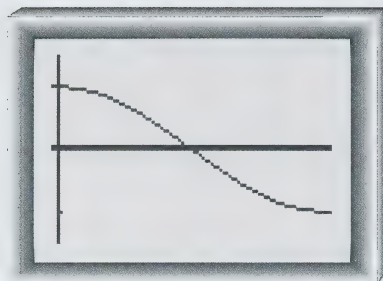


3. Use the same WINDOW settings as in textbook exercise 3, Investigation 1, page 341.

Activity 3 (continued)

4. The graph displayed on your calculator screen should look like the following.

Y= COS (X,T,Θ,n)) GRAPH



Note: Clear any entries in the

Y= screen before entering the new formula.

5. You should have made a table similar to the one in textbook exercise 5, Investigation 1, page 342. Instead of $y = \sin x$ in column 2, you should have $y = \cos x$. In the third column, you are identifying a value for x that results in the negative value of y .
6. Your table should look like the following.

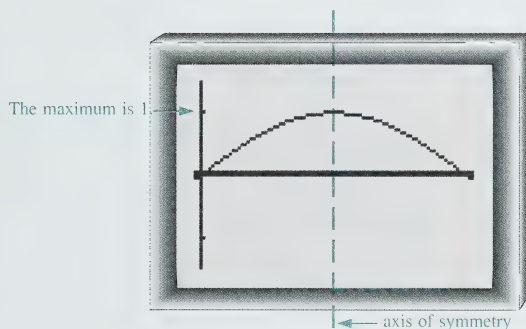
angle x	$y = \cos x$	Value for x that results in the negative value of y
2°	0.9994	178°
10°	0.9848	170°
28°	0.8829	152°
32°	0.8480	148°
46°	0.6947	134°
58°	0.5299	122°
62°	0.4695	118°
80°	0.1736	100°
88°	0.0349	92°

7. The cosine of the value in the first column is the same as the negative of the cosine of the value in the last column. The sum of the two angles is 180° .
8. The completed equations should look like the following:

$$\begin{array}{ll} \cos 2^\circ = -\cos 178^\circ & \cos 146^\circ = -\cos 34^\circ \\ \cos 10^\circ = -\cos 170^\circ & \cos 154^\circ = -\cos 26^\circ \\ \cos 38^\circ = -\cos 142^\circ & \cos 160^\circ = -\cos 20^\circ \\ \cos 66^\circ = -\cos 114^\circ & \cos 178^\circ = -\cos 2^\circ \end{array}$$

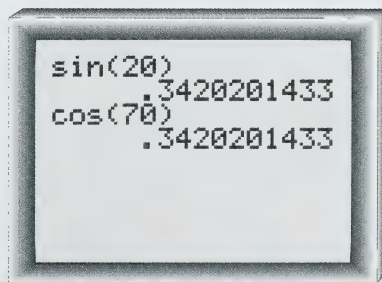
3. Textbook exercises 1, 3, 4, and 5 of “Discussing the Ideas,” p. 343

1. The graph $y = \sin x$ is symmetrical with respect to a vertical line drawn halfway between the start and end of the graph. The maximum value of $\sin x$ is 1.



3. The sine of x is $\sin(180^\circ - x)$ for any x between 0° and 180° .
4. The cosine of x is $-\cos(180^\circ - x)$ for any x between 0° and 180° .
5. The relation is $\sin x = \cos(90^\circ - x)$. An example follows.

$$\begin{aligned} \sin 20^\circ &= \cos(90^\circ - 20^\circ) \\ &= \cos 70^\circ \\ 0.3420 &= 0.3420 \end{aligned}$$



Activity 3 (continued)

4. Textbook exercises 1, 2, and 4 of “Exercises: Checking Your Skills,” p. 343

1. a. 160° b. 80° c. 133° d. 62°

2. a. The angle supplementary to 10° is 170° .

$$\sin 170^\circ = 0.1736$$

b. The angle supplementary to 123° is 57° .

$$\cos 57^\circ = 0.5446$$

c. The angle supplementary to 174° is 6° .

$$\sin 6^\circ = 0.1045$$

d. The angle supplementary to 72° is 108° .

$$\cos 108^\circ = -0.3090$$

4. If $\cos A = -0.1234$ and $0^\circ \leq A \leq 180^\circ$, you know that the measure of $\angle A$ is between 90° and 180° .

5. Textbook exercises 3.a. to 3.f., 5.a., 5.f., 5.g., and 5.i. of “Exercises: Checking Your Skills,” pp. 343 and 344

3. a. $\sin 50^\circ = 0.7660$ $\cos 50^\circ = 0.6428$

b. $\sin 130^\circ = 0.7660$ $\cos 130^\circ = -0.6428$

c. $\sin 127^\circ = 0.7986$ $\cos 127^\circ = -0.6018$

d. $\sin 140^\circ = 0.6428$ $\cos 140^\circ = -0.7660$

e. $\sin 92^\circ = 0.9994$ $\cos 92^\circ = -0.3490$

f. $\sin 40^\circ = 0.6428$ $\cos 40^\circ = 0.7660$

5. a. $\sin A = \sin 130^\circ$

$$\sin (180^\circ - 130^\circ) = \sin 130^\circ$$

$$\sin 50^\circ = \sin 130^\circ$$

$$\therefore \angle A = 50^\circ \text{ or } 130^\circ$$

f. $\sin A = \frac{1}{2}$ or 0.5000

$$\sin 30^\circ = 0.5000 \text{ and } \sin (180^\circ - 30^\circ) = 0.5000$$

$$\sin 150^\circ = 0.5000$$

$$\therefore \angle A = 30^\circ \text{ or } 150^\circ$$

Use the inverse function of sine to find the angle.

g. $\cos A = 0.8910$

$$\cos 27^\circ = 0.8910 \text{ and } \cos (180^\circ - 27^\circ) = -0.8910$$

$$\cos 153^\circ = -0.8910$$

$$\therefore \angle A = 27^\circ$$

Use the inverse function of cosine to find the angle.

i. $\cos A = -\frac{1}{2}$ or -0.5000

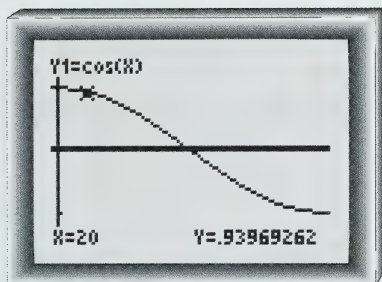
$$\cos 120^\circ = -0.5000 \text{ and } \cos (180^\circ - 120^\circ) = 0.5000$$

$$\cos 60^\circ = 0.5000$$

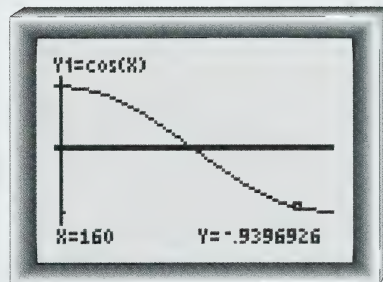
$$\therefore \angle A = 120^\circ$$

6. Textbook exercise “Communicating the Ideas,” p. 345

The cosine of an acute angle is equal to the negative of the cosine of the supplement of the acute angle. An example follows.



$$\cos 20^\circ = -\cos 160^\circ$$



Activity 4: The Sine Law

1. The assumption made is that AB is not perpendicular to the bank.
2. The length of the bridge is not the shortest distance across the river.
3. Answers will vary. Some reasons for the bridge being built at these points may include the following:
 - These points may be more convenient for building than points that are directly across.
 - These points may be on sturdier, more stable ground.
 - The points may link directly to major arterial roads.
 - There could be major environmental issues in other areas.
4. Triangle ABC is an oblique triangle; it does not contain a right angle.
5. Each angle is named using an upper-case letter; the same letter is used in lower case for the side opposite the angle.

Activity 4 (continued)

6. a. Textbook exercises 1 to 5 of “Investigation 1: The Sine Law,” pp. 347 and 348

- Measurements are not required at this point. Draw the triangles neatly and accurately. Use a sharp pencil. Label the sides of each triangle with the lower case of the letter of the angle opposite.
- Your blank table should look like the chart on page 348 of the textbook or like the one following, which is a spreadsheet.

Triangle	a	b	c	A	B	C	a/sin A	b/sin B	c/sin C	sin A/a	sin B/b	sin C/c
1												
2												
3												
4												

- Measure each side to the nearest 0.1 cm and each angle to the nearest degree. A sample is given.

Triangle	a	b	c	A	B	C	a/sin A	b/sin B	c/sin C	sin A/a	sin B/b	sin C/c
1	5.1	9.5	8	33	90	57						
2	7.0	6.0	6.4	70	52	58						
3	4	12	9	15	130	35						
4	4.8	6	3.9	54	85	41						

- Your completed table should look similar to the following.

Triangle	a	b	c	A	B	C	a/sin A	b/sin B	c/sin C	sin A/a	sin B/b	sin C/c
1	5.1	9.5	8	33	90	57	9.36	9.50	9.54	0.11	0.11	0.11
2	7.0	6.0	6.4	70	52	58	7.45	7.61	7.55	0.13	0.13	0.13
3	4	12	9	15	130	35	15.45	15.66	15.69	0.06	0.06	0.06
4	4.8	6	3.9	54	85	41	5.93	6.02	5.94	0.17	0.17	0.17

- The ratios of the side opposite the angle to the angle and vice versa are approximately the same for each of the sides and corresponding angle in a given triangle.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{and} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

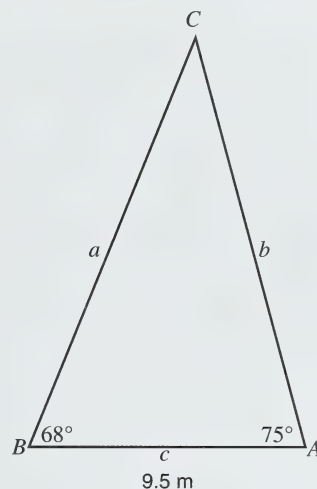
Note: Completing 6.b. is optional.

b. Textbook exercises 1 to 5 of “Investigation 2: The Sine Law Using Technology,” pp. 348 and 349

1. You should have drawn four triangles using your geometry software.
2. Measure each side to the nearest 0.1 cm and each angle to the nearest degree.
3. Use a spreadsheet to record your measurements. You need to set up a table similar to the one in exercise 3 of Investigation 1.
4. Your completed table should look similar to the one in exercise 4 of Investigation 1. Use **Copy** and **Paste** and **Fill** and **Down** to enter the formulas in the table. Follow the given instructions in textbook exercise 3 on page 349 or consult your software manual.
5. The ratios of the side opposite the angle to the angle and vice versa are approximately the same for each of the sides and corresponding angle in a given triangle.

7. Textbook exercises 1 to 5 of “Investigation 3: Using the Sine Law,” pp. 349 and 350

- 1. and 2.** Use a protractor and ruler to draw the diagram. Your diagram should look similar to the one in the textbook. Label your diagram as shown to the right.



$$\begin{aligned} 3. \quad \angle C &= 180^\circ - (\angle B + \angle A) \\ &= 180^\circ - (68^\circ + 75^\circ) \\ &= 180^\circ - 143^\circ \\ &= 37^\circ \end{aligned}$$

The measure of $\angle C$ is 37° .

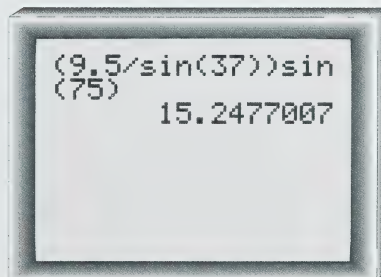
4. The required pair of ratios is $\frac{a}{\sin A} = \frac{c}{\sin C}$.

$$5. \quad \frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 75^\circ} = \frac{9.5}{\sin 37^\circ}$$

← Substitute known values.

$$\begin{aligned} a &= \left(\frac{9.5}{\sin 37^\circ} \right) \sin 75^\circ \quad \leftarrow \text{Solve for } a. \\ &\doteq 15.2 \end{aligned}$$



To the nearest 0.1 m, the length of the bridge is 15.2 m.

Activity 4 (continued)

8. Textbook exercises 1, 3, and 5 of “Discussing the Ideas,” p. 350

$$1. \quad \frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z} \quad \text{and} \quad \frac{\sin X}{x} = \frac{\sin Y}{y} = \frac{\sin Z}{z}$$

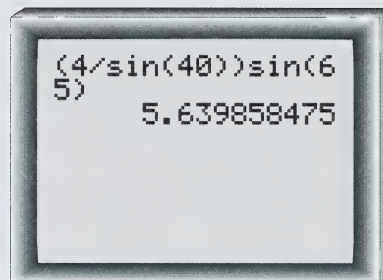
3. The given sides and angles determine which pair of the equations you will use.

5. You must have one angle and two sides or one side and two angles.

9. Textbook exercises 1 and 2.a. of “Exercises: Checking Your Skills,” p. 355

$$\begin{aligned} 1. \quad \text{a.} \quad & \frac{c}{\sin C} = \frac{a}{\sin A} \\ & \frac{c}{\sin 65^\circ} = \frac{4}{\sin 40^\circ} \\ & c = \left(\frac{4}{\sin 40^\circ} \right) \sin 65^\circ \\ & \doteq 5.6 \end{aligned}$$

The length of side c is about 5.6 units.



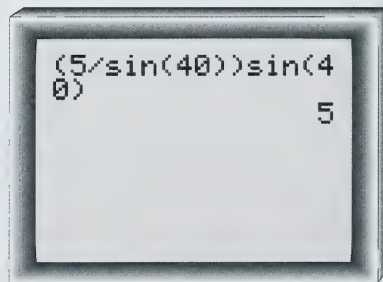
b. Step 1: Since you want to find side c , you need to know $\angle C$. Use the angle sum property.

$$\begin{aligned} \angle C &= 180^\circ - (\angle A + \angle B) \\ &= 180^\circ - (40^\circ + 100^\circ) \\ &= 180^\circ - 140^\circ \\ &= 40^\circ \end{aligned}$$

Step 2: Use the Sine Law to find side c .

$$\begin{aligned} & \frac{c}{\sin C} = \frac{a}{\sin A} \\ & \frac{c}{\sin 40^\circ} = \frac{5}{\sin 40^\circ} \\ & c = \left(\frac{5}{\sin 40^\circ} \right) \sin 40^\circ \\ & = 5 \end{aligned}$$

The length of side c is 5 units.



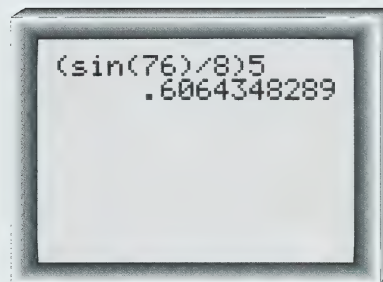
2. a. **Step 1:** Find $\sin C$.

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin C}{5} = \frac{\sin 76^\circ}{8}$$

$$\sin C = \left(\frac{\sin 76^\circ}{8} \right) 5$$

$$\doteq 0.6064$$

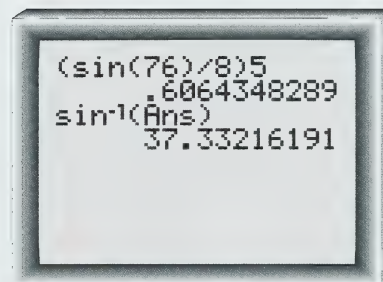
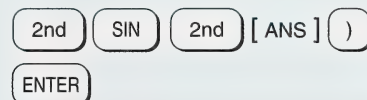


Do **not** clear the calculator screen.

- Step 2:** Use the inverse function of sine to find the measure of $\angle C$.

$$\angle C \doteq 37^\circ$$

The measure of $\angle C$ is about 37° .



10. Textbook exercises 1 and 2 of “Discussing the Ideas,” p. 355

- The particular form of the Sine Law $\left(\frac{\sin B}{b} = \frac{\sin C}{c} \right)$ was used because $\angle B$ had to be determined, and the length of side b , $\angle C$, and the length of side c were known. Using this form allows $\sin B$ (the unknown) to be on the left side of the equation.
 - The other combination of sides and angles that will work with the Sine Law is knowing two sides and an angle opposite one of the known sides.
11. It is easier to set up the solution from the equation when the quantity you want to solve for is in the numerator.

Activity 4 (continued)

12. Textbook exercises 4, 6, 7, and 8 of “Exercises: Checking Your Skills,” pp. 356 to 358

4. Draw and label a diagram to illustrate the problem. Make a plan.
The length of the bridge is represented by side c . You know side a . Find the measure of $\angle A$, and then use the Sine Law to determine the length of side c .

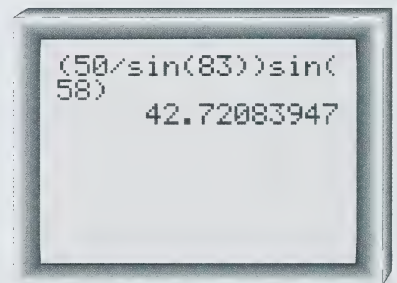
Step 1: Use the angle sum property to find the measure of $\angle A$.

$$\begin{aligned}\angle A &= 180^\circ - (\angle B + \angle C) \\ &= 180^\circ - (39^\circ + 58^\circ) \\ &= 180^\circ - 97^\circ \\ &= 83^\circ\end{aligned}$$

The measure of $\angle A$ is 83° .

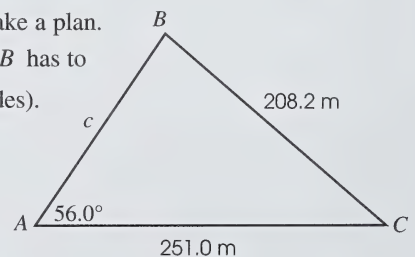
Step 2: Find the length of side c . Use the pair of ratios from the Sine Law where you know three of the four quantities. To make solving easier, the unknown should be in the numerator position on the left side of the equation.

$$\begin{aligned}\frac{c}{\sin C} &= \frac{a}{\sin A} \\ \frac{c}{\sin 58^\circ} &= \frac{50}{\sin 83^\circ} \\ c &= \left(\frac{50}{\sin 83^\circ} \right) \sin 58^\circ \\ &\doteq 42.72\end{aligned}$$



The length of the bridge is about 42.7 m.

6. a. Draw and label a diagram to illustrate the problem. Make a plan.
The triangle has been labelled ABC ; the measure of $\angle B$ has to be found ($\angle B$ is the angle between the shorter two sides).



$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{251.0} = \frac{\sin 56.0^\circ}{208.2}$$

$$\sin B = \left(\frac{\sin 56.0^\circ}{208.2} \right) \times 251.0$$

$$\doteq 0.9995$$

Do **not** clear the calculator.

Use the inverse function of sine to find the measure of $\angle B$.

$$\angle B \doteq 88.12^\circ$$

The measure of $\angle B$ is about 88.1° .

```
(sin(56)/208.2)*
251
.9994641245
```

```
(sin(56)/208.2)*
251
.9994641245
sin-1(Ans)
88.12418936
```

- b. You determined the measure of $\angle B$ in part a. Now that you know the measures of $\angle A$ and $\angle B$, you can find the measure of $\angle C$, and then use the Sine Law to find the length of side c , the shortest side.

Step 1: Use the angle sum property to find the measure of $\angle C$.

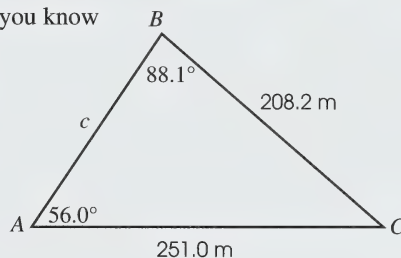
$$\angle C = 180^\circ - (\angle A + \angle B)$$

$$\doteq 180^\circ - (56.0^\circ + 88.1^\circ)$$

$$\doteq 180^\circ - 144.1^\circ$$

$$\doteq 35.9^\circ$$

The measure of $\angle C$ is 35.9° .



Activity 4 (continued)

Step 2: To find the measure of side c (the shortest side), you can use either of two equations involving the Sine Law since you now know the lengths of both of the other two sides and their opposite angles.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{c}{\sin 35.9^\circ} = \frac{208.2}{\sin 56.0^\circ}$$

$$c = \frac{208.2}{\sin 56.0^\circ} \times \sin 35.9^\circ$$

$$\approx 147.26$$

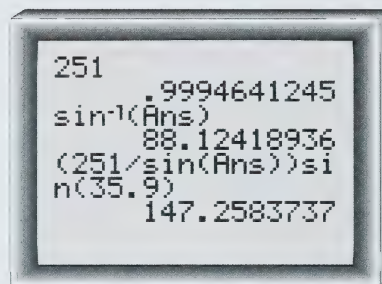
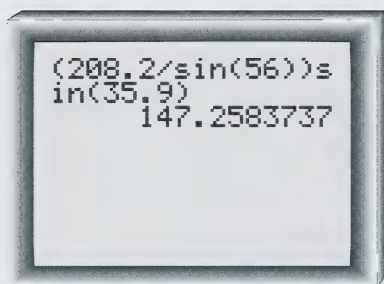
or

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{c}{\sin 35.9^\circ} = \frac{251.0}{\sin 88.1^\circ}$$

$$c = \frac{251.0}{\sin 88.1^\circ} \times \sin 35.9^\circ$$

$$\approx 147.26$$



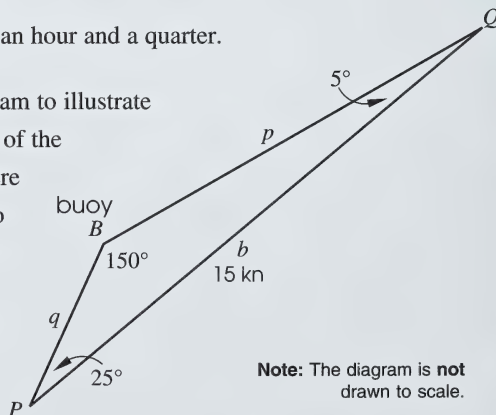
The length of the shortest side, side c , is about 147.3 m.

7. **Step 1:** Determine the distance the boat travelled in an hour and a quarter.

$$\begin{aligned} \text{distance} &= \text{speed} \times \text{time} \\ &= 12 \times 1.25 \\ &= 15 \end{aligned}$$

The boat travelled 15 kn (nautical miles) in an hour and a quarter.

Step 2: Clarify the problem. Draw and label a diagram to illustrate the situation. (Use the diagram on page 357 of the textbook for guidance.) Label the point where the buoy is located point B . Make a plan. To determine the distance from the boat to the buoy (side p), you need to know the measure of the angle at P ($\angle BPQ$) and the measure of the angle at B ($\angle PBQ$).



$$\begin{aligned}\angle BPQ &= 45^\circ - 20^\circ \\ &= 25^\circ\end{aligned}$$

$$\begin{aligned}\angle BQP &= 230^\circ - (180^\circ + 45^\circ) \\ &= 230^\circ - 225^\circ \\ &= 5^\circ\end{aligned}$$

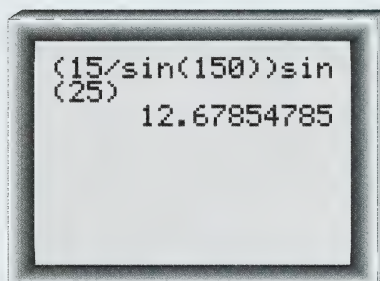
$$\begin{aligned}\angle PBQ &= 180^\circ - (\angle BPQ + \angle PQB) \\ &= 180^\circ - (25^\circ + 5^\circ) \\ &= 180^\circ - 30^\circ \\ &= 150^\circ\end{aligned}$$

← angle sum property

Use three letters to name an angle when there is more than one angle around a point.

Use the Sine Law to find the distance from the boat to the buoy.

$$\begin{aligned}\frac{p}{\sin BPQ} &= \frac{b}{\sin PBQ} \\ \frac{p}{\sin 25^\circ} &= \frac{15}{\sin 150^\circ} \\ p &= \left(\frac{15}{\sin 150^\circ} \right) \sin 25^\circ \\ &\doteq 12.7\end{aligned}$$

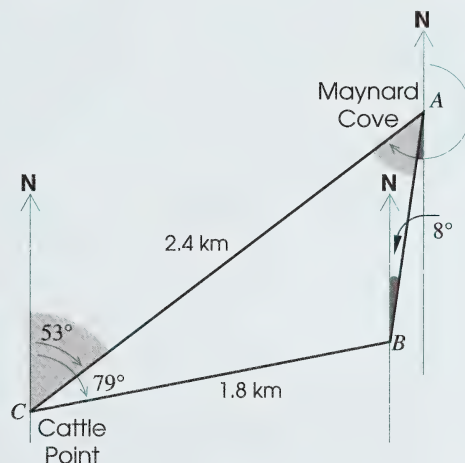


The distance from the boat to the buoy is about 12.7 kn (nautical miles).

8. a. Draw and label a diagram. Clarify the problem. Cattle Point is in the opposite direction from Maynard Cove so the bearing of Cattle Point is equal to the bearing of Maynard Cove from Cattle Point plus 180° . The shaded angles are alternate angles; they are angles on opposite sides of a transversal that cuts the two parallel lines.

$$\begin{aligned}\text{bearing} &= 53^\circ + 180^\circ \\ &= 233^\circ\end{aligned}$$

The bearing of Cattle Point from Maynard Cove is 233° .



Note: The diagram is **not** drawn to scale.

Activity 4 (continued)

- b. Draw and label a diagram to illustrate the problem. Make a plan. Notice the two pairs of alternate angles. Using the information from part a., you can make the following calculations:

$$\angle ACB = 79^\circ - 53^\circ$$

$$= 26^\circ$$

$$\angle BAC = 53^\circ - 8^\circ$$

$$= 45^\circ$$

The total distance the boat travels from Cattle Point to Maynard Cove is the sum of the length of AB (or side c) and the length of BC (or side a , 1.8 km).

Find the length of AB (side c). Use the pair of ratios from the Sine Law where you know three of the four quantities.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{c}{\sin 26^\circ} = \frac{1.8}{\sin 45^\circ}$$

$$c = \left(\frac{1.8}{\sin 45^\circ} \right) \sin 26^\circ$$

$$\doteq 1.12$$

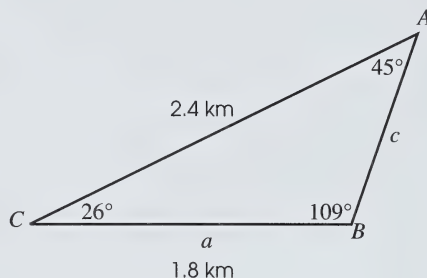
Find the total distance travelled.

$$\text{total distance} = AB + BC$$

$$\doteq 1.12 + 1.8$$

$$\doteq 2.92$$

The total distance the boat travels is about 2.9 km.



```
(1.8/sin(45))sin
(26)
1.115910758
```

```
(1.8/sin(45))sin
(26)
1.115910758
Ans+1.8
2.915910758
```

13. Textbook exercise “Communicating the Ideas,” p. 350

The Sine Law works like any ratio. To solve a ratio, you cross-multiply and solve for the unknown. If you have two unknowns on one side or an unknown on each side of the equal sign, you cannot solve the equality.

Activity 5: The Cosine Law

1. Textbook exercises 1 to 5 of “Discussing the Ideas,” p. 364

1. $p^2 = q^2 + r^2 - 2qr \cos P$ $q^2 = p^2 + r^2 - 2pr \cos Q$ $r^2 = p^2 + q^2 - 2pq \cos R$
2. To determine a side using the Cosine Law, you must know the measures of two of the sides and the included angle.
3. To determine an angle using the Cosine Law, you must know the measures of the three sides of the triangle.
4. The smallest angle of a triangle is opposite the shortest side.
5. You can determine the measures of the two remaining angles by using these methods:
 - **Method 1:** Use the Cosine Law to determine one of the other angles; then, use the angle sum property—subtract the sum of the two known angles from 180° to find the third angle.
 - **Method 2:** Use the Sine Law to determine one of the other angles; then, use the angle sum property—subtract the sum of the two known angles from 180° to find the third angle.
 - **Method 3:** Use the Cosine Law to determine the remaining angles.
 - **Method 4:** Use the Sine Law to determine the remaining angles.

2. Textbook exercise 1 of “Exercises: Checking Your Skills,” p. 365

1. The length of AC in each triangle is the length of side b . Different methods of calculation are shown in each of view a., view b., and view c.

a. $b^2 = a^2 + c^2 - 2ac \cos B$

$$b^2 = 4^2 + 3^2 - 2(4)(3) \cos 60^\circ$$

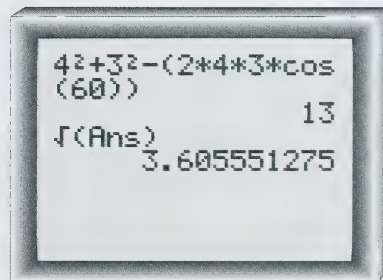
$$b^2 = 16 + 9 - 24 \cos 60^\circ$$

$$b^2 = 25 - 24 \cos 60^\circ$$

$$b = \sqrt{25 - 24 \cos 60^\circ}$$

$$\approx 3.61$$

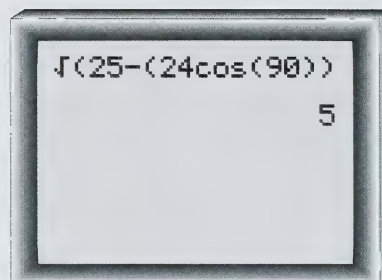
The length of AC is about 3.6 units.



Activity 5 (continued)

b. $b^2 = a^2 + c^2 - 2ac \cos B$
 $b^2 = 4^2 + 3^2 - 2(4)(3) \cos 90^\circ$
 $b^2 = 16 + 9 - 24 \cos 90^\circ$
 $b^2 = 25 - 24 \cos 90^\circ$
 $b = \sqrt{25 - 24 \cos 90^\circ}$
 $= 5$

2nd [$\sqrt{\quad}$] 2 5 - (2 4 COS 9
 0)) ENTER

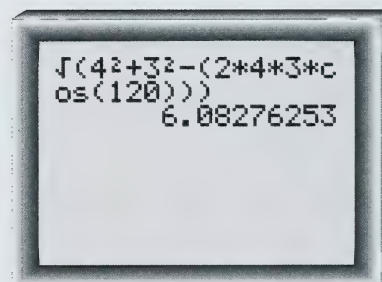


The length of AC is 5 units.

Note: You could use the Pythagorean Theorem $(a^2 + b^2 = c^2)$ here since this is a right triangle.

c. $b^2 = a^2 + c^2 - 2ac \cos B$
 $b^2 = 4^2 + 3^2 - 2(4)(3) \cos 120^\circ$
 $b^2 = 16 + 9 - 24 \cos 120^\circ$
 $b^2 = 25 - 24 \cos 120^\circ$
 $b = \sqrt{25 - 24 \cos 120^\circ}$
 ≈ 6.08

2nd [$\sqrt{\quad}$] 4 x^2 + 3 x^2 - (2 $\times$ 4 $\times$ 3 $\times$ COS 1 2 0)
)) ENTER



The length of AC is about 6.1 units.

3. Textbook exercise 2 of "Exercises: Checking Your Skills," p. 365

2. a. Step 1: Choose the appropriate Cosine Law.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Step 2: Substitute the known values. Solve for the unknown.

$$8^2 = 6^2 + 6^2 - 2(6)(6) \cos A$$

$$64 = 36 + 36 - 72 \cos A$$

$$64 = 72 - 72 \cos A$$

$$-8 = -72 \cos A$$

$$72 \cos A = 8$$

$$\cos A = \frac{8}{72}$$

$$\cos A = \frac{1}{9} \quad \text{or} \quad 0.1111$$

Calculator screen showing the calculation of $\cos A$:
 $(8^2 - 6^2 - 6^2) / (-2 * 6 * 6)$
 $.1111111111$

Step 3: Use the inverse function of cosine to find the measure of $\angle A$.

$$\angle A \doteq 83.62^\circ$$

The measure of $\angle A$ is about 83.6° .

Calculator screen showing the calculation of $\angle A$:
 $(8^2 - 6^2 - 6^2) / (-2 * 6 * 6)$
 $.1111111111$
 $\cos^{-1}(\text{Ans})$
 83.62062979

b. $a^2 = b^2 + c^2 - 2bc \cos A$

$$6^2 = 8^2 + 5^2 - 2(8)(5) \cos A$$

$$6^2 - 8^2 - 5^2 = -2(8)(5) \cos A$$

$$\frac{6^2 - 8^2 - 5^2}{-2(8)(5)} = \cos A$$

$$\cos A = 0.6625$$

$$\angle A \doteq 48.51^\circ$$

Calculator screen showing the calculation of $\angle A$:
 $(6^2 - 8^2 - 5^2) / (-2 * 8 * 5)$
 $.6625$
 $\cos^{-1}(\text{Ans})$
 48.50918314

The measure of $\angle A$ is about 48.5° .

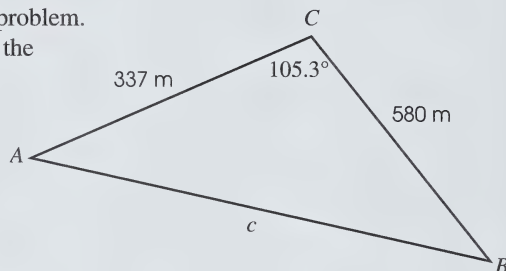
Activity 5 (continued)

4. Textbook exercises 3 and 10 of “Exercises: Checking Your Skills,” pp. 365 and 367

3. **Step 1:** Draw and label a diagram to illustrate the problem.

The distance between pole A and pole B is the length of AB , side c .

- Step 2:** Make a plan to solve the problem. You know two sides and the included angle. Use the Cosine Law with AB , side c , isolated.



$$c^2 = a^2 + b^2 - 2ab \cos C$$

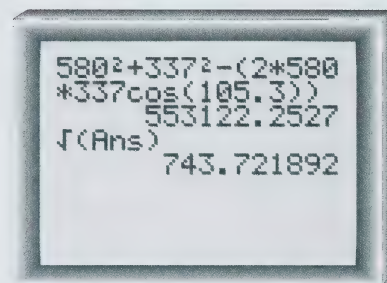
- Step 3:** Substitute known values and solve.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 580^2 + 337^2 - 2(580)(337) \cos 105.3^\circ$$

$$c^2 = 553\,122.2527$$

$$c \doteq 743.7$$

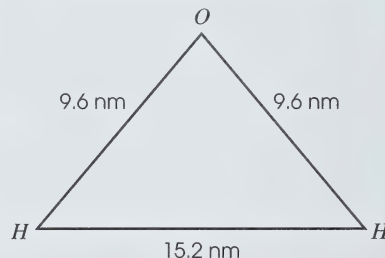


The distance between pole A and pole B is about 744 m.

10. **Step 1:** Draw and label a diagram to illustrate the problem.

Use letters on the diagram to represent the atoms.

You need to find the measure of $\angle O$.



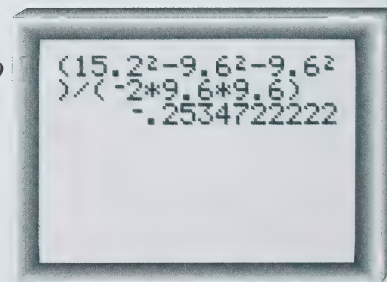
- Step 2:** Make a plan. Use the Cosine Law with side o isolated. Solve for $\cos O$.

$$o^2 = h^2 + h^2 - 2(h)(h) \cos O$$

$$15.2^2 = 9.6^2 + 9.6^2 - 2(9.6)(9.6) \cos O$$

$$\frac{15.2^2 - 9.6^2 - 9.6^2}{-2(9.6)(9.6)} = \cos O$$

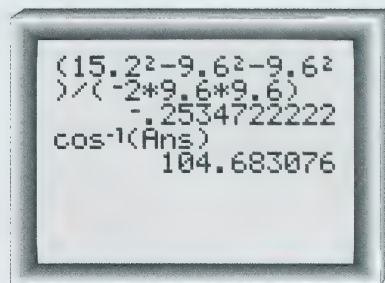
$$-0.2535 \doteq \cos O$$



Step 3: Use the inverse function of cosine to find the measure of $\angle O$.

$$\angle O \doteq 104.683^\circ$$

The bond angle of the water molecule is about 104.68° .



5. Methods may vary; for example, you may choose to find all the angles first and then determine the bearings.

Step 1: Determine the bearing of path AB .

Since path AB is along the north line and in the opposite direction of north, the angle between the north line and the path is 180° . Therefore, the bearing of path AB is 180° .

Step 2: Find the angle between paths AB and AC at $\angle A$. Use the Cosine Law with side a isolated.

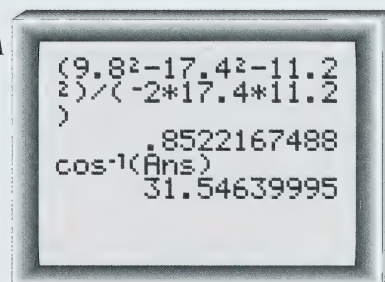
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$9.8^2 = 17.4^2 + 11.2^2 - 2(17.4)(11.2) \cos A$$

$$\frac{9.8^2 - 17.4^2 - 11.2^2}{-2(17.4)(11.2)} = \cos A$$

$$0.8522 \doteq \cos A$$

$$\angle A \doteq 31.5^\circ$$



The angle at A is about 31.5° .

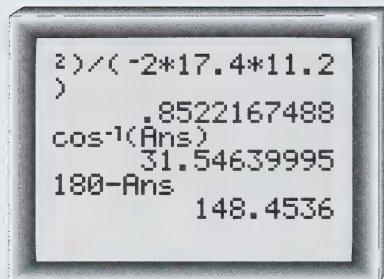
Do **not** clear the calculator.

Activity 5 (continued)

Step 3: Determine the bearing of path AC .

$$\text{bearing} = 180^\circ - \angle A$$

$$\doteq 148^\circ \quad \leftarrow \text{Round to 3 digits.}$$



The bearing of path AC is about 148° .

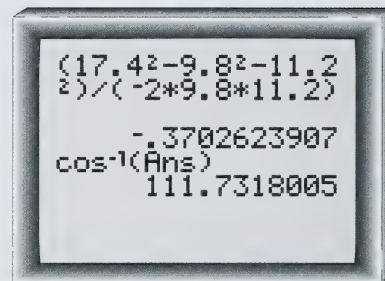
Step 4: Determine the angle between paths AB and BC at B . Use the Cosine Law with side b isolated.

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$17.4^2 = 9.8^2 + 11.2^2 - 2(9.8)(11.2) \cos B$$

$$\frac{17.4^2 - 9.8^2 - 11.2^2}{-2(9.8)(11.2)} = \cos B$$

$$\angle B \doteq 111.73^\circ$$

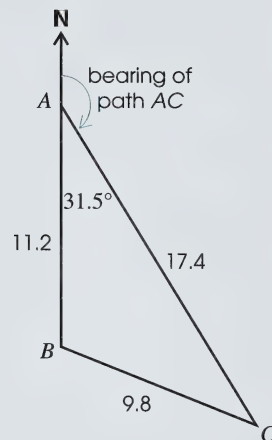


The angle at B is about 111.7° .

Do **not** clear the calculator.

Step 5: Determine the bearing of path BC .

Since AB is the north line, the bearing of BC is about 112° (the measure of $\angle B$).

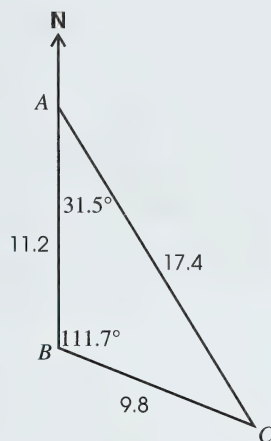


Step 6: Find the angle between paths AC and BC at C . You may use the angle sum property or the Sine Law or the Cosine Law.

$$\begin{aligned}\angle C &= 180^\circ - (\angle A + \angle B) \\ &\doteq 180^\circ - (31.55^\circ + 111.73^\circ) \\ &\doteq 180^\circ - 143.28^\circ \\ &\doteq 36.72^\circ\end{aligned}$$

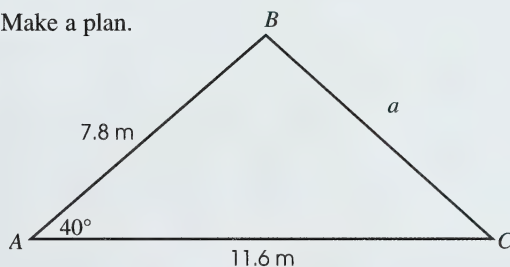
The simplest method is using the angle sum property.

The angle at C is about 36.8° .



6. Step 1: Draw and label a diagram to illustrate the problem. Make a plan.

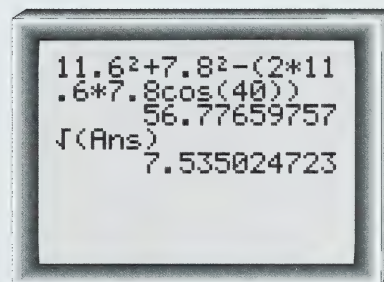
You need to find the length of side a , the measure of $\angle B$, and the measure of $\angle C$.



Step 2: You know the lengths of two sides and the included angle. Find the length of the steepest roof truss, side a , by using the Cosine Law with side a isolated.

$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cos A \\ a^2 &= 11.6^2 + 7.8^2 - 2(11.6)(7.8) \cos 40^\circ \\ a^2 &\doteq 56.77 \\ a &\doteq \sqrt{56.77} \\ &\doteq 7.54\end{aligned}$$

The length of the steepest roof truss is about 7.5 m.



Do **not** clear your calculator.

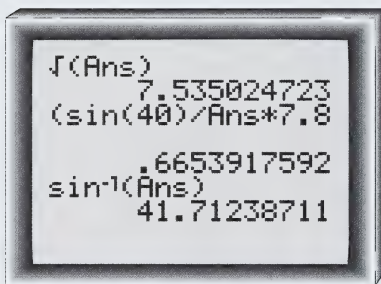
Activity 5 (continued)

Step 3: Find the measure of the angle at C . You can use the Sine Law or the Cosine Law.

Using the Sine Law

$$\begin{aligned}\frac{\sin A}{a} &= \frac{\sin C}{c} \\ \frac{\sin 40^\circ}{7.54} &\doteq \frac{\sin C}{7.8} \\ \sin C &\doteq \left(\frac{\sin 40^\circ}{7.54} \right) 7.8 \\ \sin C &\doteq 0.6654 \\ \angle C &\doteq 41.7^\circ\end{aligned}$$

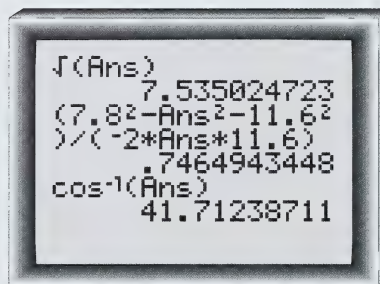
Angle C is about 42°



Using the Cosine Law

$$\begin{aligned}c^2 &= a^2 + b^2 - 2ab \cos C \\ 7.8^2 &\doteq 7.5^2 + 11.6^2 - 2(7.5)(11.6) \cos C \\ \frac{7.8^2 - 7.5^2 - 11.6^2}{-2(7.5)(11.6)} &\doteq \cos C \\ 0.7465 &\doteq \cos C \\ \angle C &\doteq 41.7^\circ\end{aligned}$$

Angle C is about 42° .



Step 4: Find the measure of $\angle B$ by using the angle sum property or the Cosine Law.

Using the Angle Sum Property

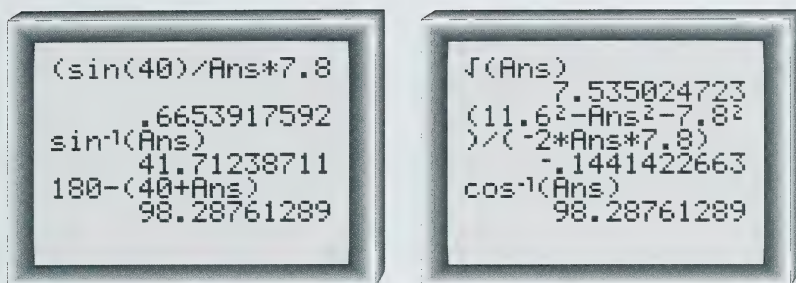
$$\begin{aligned}\angle B &= 180^\circ - (\angle A + \angle C) \\ &\doteq 180^\circ - (40^\circ + 41.7^\circ) \\ &\doteq 180^\circ - 81.7^\circ \\ &\doteq 98.3^\circ\end{aligned}$$

Angle B is about 98° .

Using the Cosine Law

$$\begin{aligned}b^2 &= a^2 + c^2 - 2ac \cos B \\ 11.6^2 &\doteq 7.5^2 + 7.8^2 - 2(7.5)(7.8) \cos B \\ \frac{11.6^2 - 7.5^2 - 7.8^2}{-2(7.5)(7.8)} &\doteq \cos B \\ \angle B &\doteq 98.3^\circ\end{aligned}$$

Angle B is about 98° .



Therefore, side BC is about 7.5 m, $\angle B$ is about 98° , and $\angle C$ is about 42° .

Note: Minor differences occur in calculations if the rounded answers are used. If the original answer from Step 2 is used throughout, the answers using different methods of calculation are closer in value.

7. Textbook exercise “Communicating the Ideas,” p. 367

The Cosine Law can be thought of as a generalization of the Pythagorean Theorem since it can be used to find the third side in a right triangle.

Activity 6: Selecting a Strategy for Problem Solving

1. **a. Trigonometry** is the study of the relationships between the sides and angles of a triangle in terms of the basic trigonometric functions (sine, cosine, tangent).
- b. Right-triangle trigonometry** involves solving triangles for unknown sides and angles using the basic trigonometric ratios of sine, cosine, and tangent.
- c. Oblique triangle trigonometry** involves solving oblique triangles for unknown sides and angles using the Sine Law and Cosine Law.
- d. The angle sum property** states that the sum of the three angles in a triangle is 180° .
2. If you know two angles in a triangle, then you can find the third angle by using the angle sum property; that is, by subtracting the sum of the two known angles from 180° .
3. The following steps are suggested for solving the problem in Example 2 (pages 370 and 371 of the textbook):

Step 1: Draw and label a diagram illustrating the situation in the problem.

Step 2: Assess which information is given and what information you are required to determine. Decide which formula or formulas are required.

Step 3: Calculate each required unknown angle or side.

Activity 6 (continued)

4. Textbook exercises 1, 2, and 4 of “Discussing the Ideas,” p. 372

1. Use the Sine Law if you are given two sides and one angle or one side and two angles of a triangle. If you are given three sides or two sides and the included angle of a triangle, use the Cosine Law.
2. You will need more than one step if you need to determine an unknown angle or side before you can determine the required angle or side.
4. Answers will vary. You may have chosen the following alternative method.

Once you have determined side a using the Cosine Law, you can use the Sine Law to determine the other two angles.

5. Textbook exercises 2, 4, and 7 of “Exercises: Checking Your Skills,” pp. 372 to 374

2. **Step 1:** Draw and label a diagram to illustrate the problem.

Assess the given information and make a plan.

Side AB represents the height of Della Falls.

Side AB is part of right $\triangle ABD$. Side AD is common to $\triangle ABD$ and $\triangle ACD$.

Step 2: Angle ADC is supplementary to $\angle ADB$;

$$\angle ADC = 180^\circ - 62^\circ = 118^\circ.$$

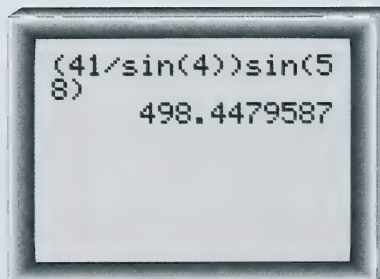
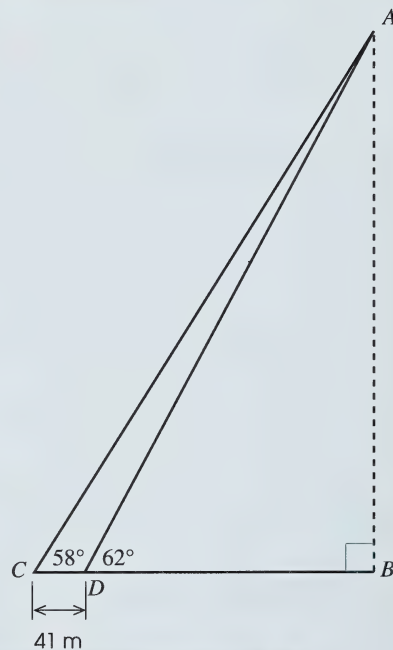
Using the angle sum property, the measure of

$$\angle CAD = 180^\circ - (118^\circ + 58^\circ) = 4^\circ.$$

Use the Sine Law to find the length of AD (side c).

$$\begin{aligned}\frac{c}{\sin C} &= \frac{a}{\sin A} \\ \frac{c}{\sin 58^\circ} &= \frac{41}{\sin 4^\circ} \\ c &= \left(\frac{41}{\sin 4^\circ} \right) \sin 58^\circ \\ &\doteq 498.4\end{aligned}$$

Do **not** clear the calculator.



Step 3: Triangle ABD is a right triangle. Use the sine ratio to find the length of AB .

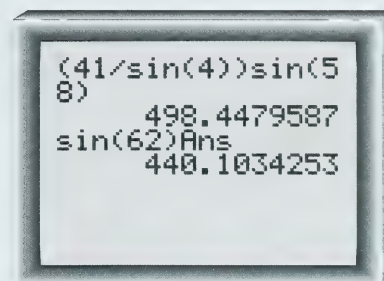
$$\sin D = \frac{\text{length of side opposite}}{\text{length of hypotenuse}}$$

$$\sin D = \frac{AB}{AD}$$

$$\sin 62^\circ = \frac{AB}{498}$$

$$498 \sin 62^\circ = AB$$

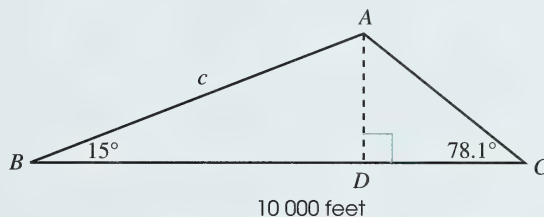
$$AB \doteq 440.1$$



Della Falls is about 440 m in height.

- 4. Step 1:** Draw and label a diagram to illustrate the problem.

Assess the given information and make a plan. The height of the clouds is represented by a perpendicular drawn from $\angle A$ to BC . This measure is illustrated in the diagram. Right $\triangle ACD$ is formed.



Note: The drawing is **not** drawn to scale.

Step 2: Use the angle sum property to find the measure of $\angle A$.

$$\begin{aligned}\angle A &= 180^\circ - (15^\circ + 78.1^\circ) \\ &= 86.9^\circ\end{aligned}$$

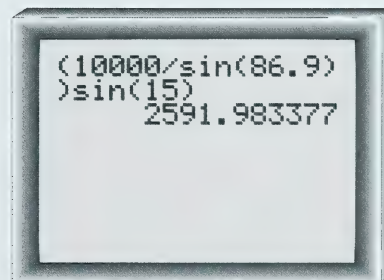
Step 3: Use the Sine Law to find the length of AC (side b).

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{b}{\sin 15^\circ} = \frac{10\,000}{\sin 86.9^\circ}$$

$$b = \left(\frac{10\,000}{\sin 86.9^\circ} \right) \sin 15^\circ$$

$$b \doteq 2592.0$$



Do **not** clear the calculator.

Activity 6 (continued)

Step 4: You know an angle and the hypotenuse of a right triangle. Use the sine ratio to find the length of AD , the side opposite $\angle C$.

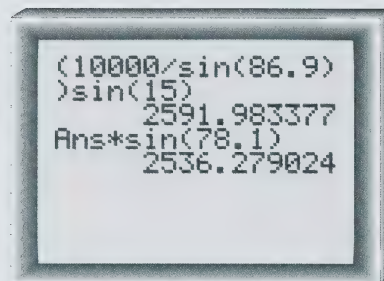
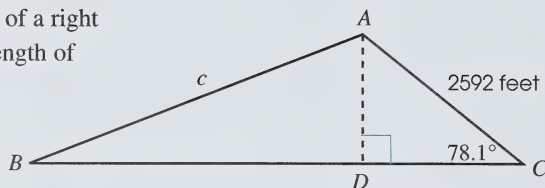
$$\sin C = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\sin C = \frac{AD}{AC}$$

$$\sin 78.1^\circ = \frac{AD}{2592}$$

$$AD = 2592 \sin 78.1^\circ$$

$$\approx 2536.3$$

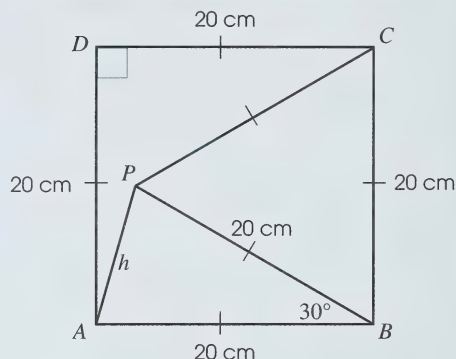


The height of the clouds is about 2536 feet.

7. Draw and label a diagram to illustrate the problem. Make a plan. Triangle PCB is an equilateral triangle—all sides are equal. That means that each angle is equal. Each angle is 60° . The length of each side is equal. Each side is 20 cm.

Angle CBA is 90° since it is a corner of a square. If $\angle CBA$ is 90° and $\angle CBP$ is 60° , then $\angle PBA$ is $90^\circ - 60^\circ = 30^\circ$.

You now know two sides and the included angle. Use the Cosine Law to find the length of h .



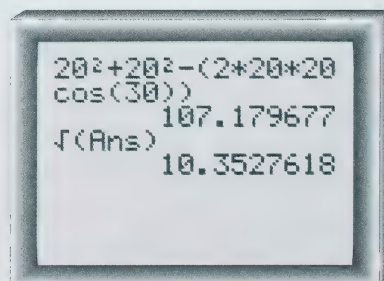
$$h^2 = a^2 + p^2 - 2ap \cos B$$

$$h^2 = 20^2 + 20^2 - 2(20)(20) \cos 30^\circ$$

$$h^2 \approx 107$$

$$h \approx \sqrt{107}$$

$$\approx 10.4$$



The length of side h is about 10 cm.

6. Textbook exercise “Communicating the Ideas,” p. 375

You could suggest that these steps be followed:

Step 1: Draw and label a diagram illustrating the situation in the problem.

Step 2: Assess which information is given and what information you are required to determine.
Decide which formula or formulas are required.

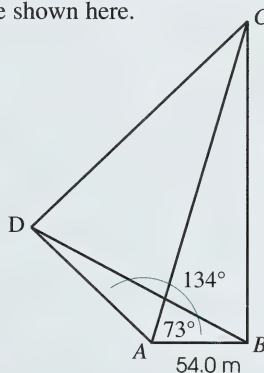
Step 3: Calculate each required unknown angle or side.

Follow-up Activities

1. Textbook exercises 1 to 7 of Part A of “What Should I Be Able To Do?” p. 379

Note: The diagrams are **not** drawn to scale.

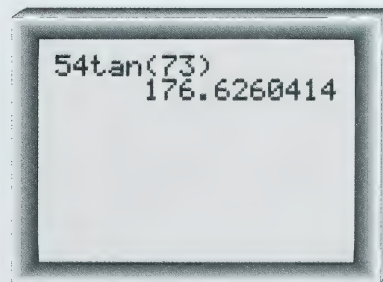
1. Your diagram should be similar to the one shown here.



2. Since $\triangle ABC$ is a right triangle, you can use the appropriate trigonometric ratio. The length of BC is the side opposite $\angle A$; side AB is the side adjacent to $\angle A$. Use the tangent ratio.

$$\begin{aligned}\tan A &= \frac{\text{side opposite}}{\text{side adjacent}} \\ \tan A &= \frac{BC}{AB} \\ \tan 73^\circ &= \frac{BC}{54.0} \\ BC &= 54.0 \tan 73^\circ \\ &\doteq 176.6\end{aligned}$$

The length of BC is about 176.6 m.



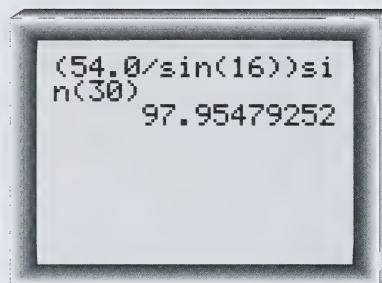
Follow-up Activities (continued)

3. You can use $\triangle BAD$ to determine the length of AD . Draw and label a diagram to illustrate the problem. Use the angle sum property to find the measure of $\angle ADB$.

$$\begin{aligned}\angle ADB &= 180^\circ - (134^\circ + 30^\circ) \\ &= 180^\circ - 164^\circ \\ &= 16^\circ\end{aligned}$$

Use the Sine Law to find the length of AD (side b).

$$\begin{aligned}\frac{b}{\sin B} &= \frac{d}{\sin D} \\ \frac{b}{\sin 30^\circ} &= \frac{54.0}{\sin 16^\circ} \\ b &= \left(\frac{54.0}{\sin 16^\circ} \right) \sin 30^\circ \\ &\doteq 98.0\end{aligned}$$

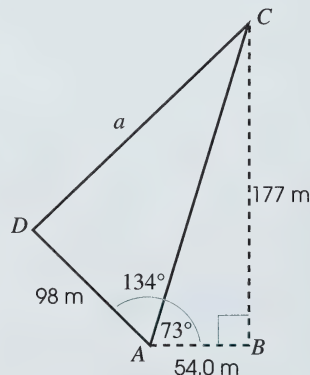


The length of AD is about 98.0 m.

4. You can use $\triangle CAD$ to find the measure of DC .

Step 1: Draw and label a diagram to illustrate the problem.

Assess the given information and make a plan.
Since $\angle BAD$ is 134° and $\angle BAC$ is 73° , $\angle CAD$ is $\angle BAD - \angle BAC = 134^\circ - 73^\circ = 61^\circ$. There is insufficient information to calculate the length of DC from the known information. You need to use information from $\triangle ABC$.



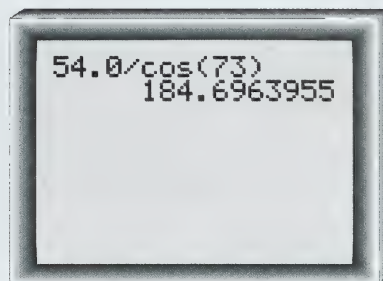
Step 2: First, find the length of AC . Since $\triangle ABC$ is a right triangle, you can use the cosine ratio to determine AC .

$$\cos BAC = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\cos BAC = \frac{AB}{AC}$$

$$AC = \frac{54.0}{\cos 73^\circ}$$

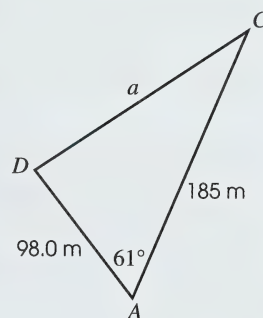
$$\doteq 184.7$$



Do **not** clear the calculator.
You will need this answer
for textbook exercise 5.

- 5. Step 1:** Draw and label a diagram to illustrate the problem.
(**Note:** You may refer to the diagram in textbook exercise 4.
Label the angles and sides you were able to determine.)

Assess the given information and make a plan. You know
two sides and the included angle.



Step 2: Use the Cosine Law to determine the unknown side. (**Note:** Use the unrounded value from textbook exercise 4 in your calculations.)

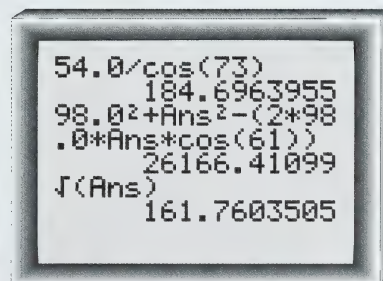
$$a^2 = c^2 + d^2 - 2cd \cos A$$

$$a^2 \doteq 98.0^2 + 184.7^2 - 2(98.0)(184.7) \cos 61^\circ$$

$$a^2 \doteq 26\,166.4$$

$$a \doteq \sqrt{26\,166.4}$$

$$\doteq 161.8$$



The length of DC is about 162 m.

Follow-up Activities (continued)

6. The perimeter of the plot of land is the sum of all the outer edges.

$$\begin{aligned}\text{Perimeter} &= AB + BC + DC + AD \\ &\doteq 54.0 + 176.6 + 161.8 + 98.0 \\ &\doteq 490.4\end{aligned}$$

The perimeter of the plot of land is about 490 m.

7. The fencing cost is \$40/m.

$$\begin{aligned}\text{Total cost} &= \text{perimeter} \times \text{cost per metre} \\ &\doteq 490.4 \times 40 \\ &\doteq 19\,616\end{aligned}$$

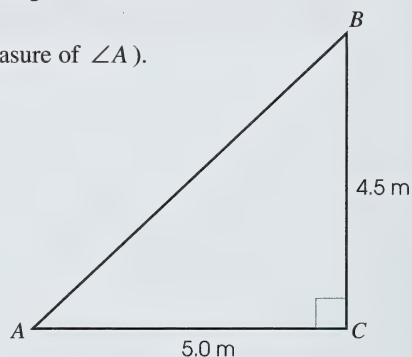
The cost to fence the plot of land is about \$19 616.

2. Textbook exercises 8, 10, 11, and 12 of Part B of “What Should I Be Able To Do?” p. 380

8. Draw and label a diagram to illustrate the problem. Assess the given information and make a plan. Points A , B , and C make a right triangle.

Use the tangent ratio to find the angle of inclination (the measure of $\angle A$).

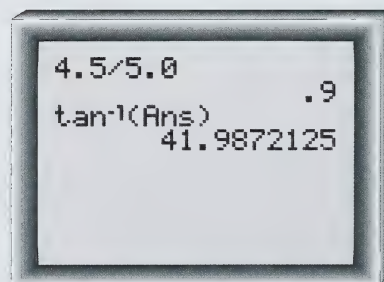
$$\begin{aligned}\tan A &= \frac{\text{side opposite}}{\text{side adjacent}} \\ &= \frac{BC}{AC} \\ &= \frac{4.5}{5.0} \\ &= 0.9000\end{aligned}$$



Use the inverse function of tangent to find the measure of $\angle A$.

$$\angle A \doteq 42.0^\circ$$

Therefore, the angle of inclination of the escalator is about 42° .



10. a. Use the inverse function of sine to find the measure of $\angle A$.

$$\begin{aligned}\sin A &= 0.7071 \\ \angle A &\doteq 45^\circ \\ \sin 45^\circ &= \sin(180^\circ - 45^\circ) \\ &= \sin 135^\circ\end{aligned}$$

Therefore, $\angle A = 45^\circ$ and 135° .

- b. Use the inverse function of cosine to find the measure of $\angle A$.

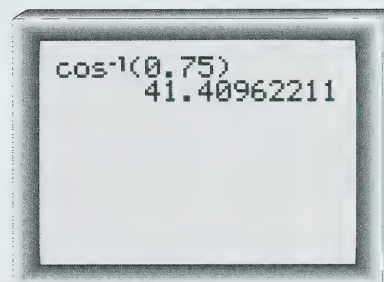
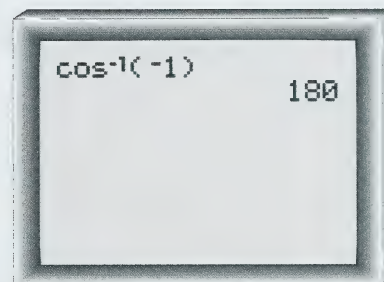
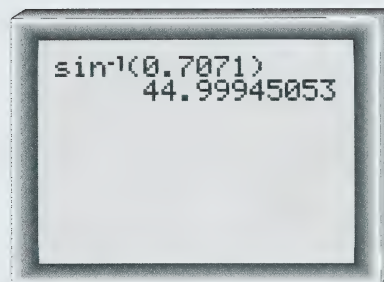
$$\begin{aligned}\cos A &= -1 \\ \angle A &= 180^\circ\end{aligned}$$

Therefore, $\angle A = 180^\circ$.

- c. Use the inverse function of cosine to find the measure of $\angle A$.

$$\begin{aligned}\cos A &= 0.75 \\ \angle A &\doteq 41.4^\circ\end{aligned}$$

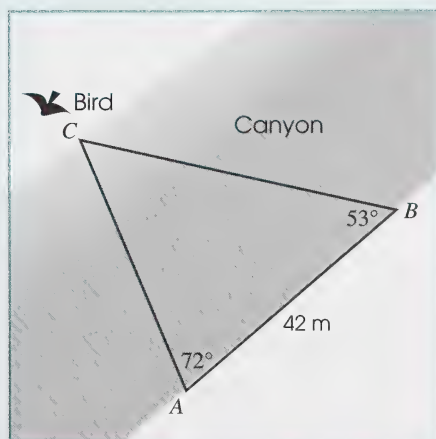
Therefore, $\angle A$ is about 41° .



11. Draw and label a diagram to illustrate the problem.

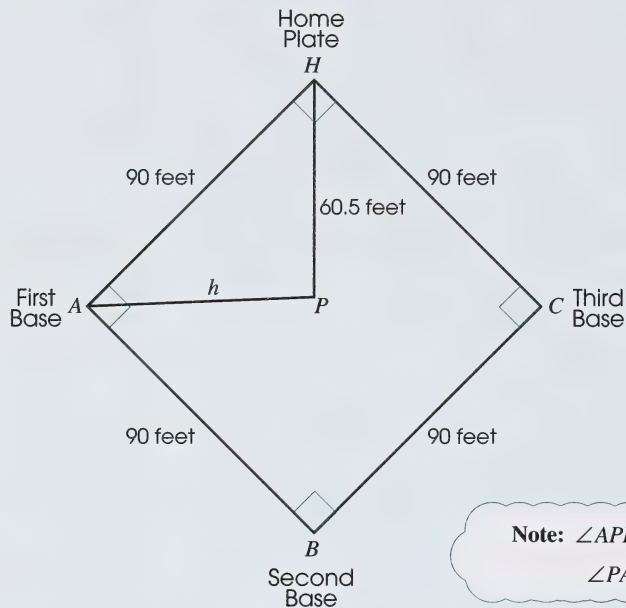
No calculation is necessary.

Friend A is closer to the bird since the shorter side (AC) is opposite the smaller angle ($53^\circ < 72^\circ$).

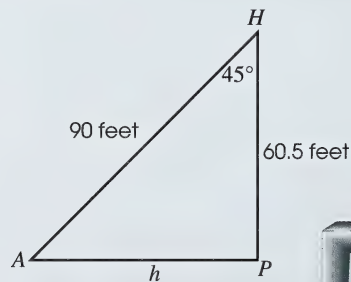


Follow-up Activities (continued)

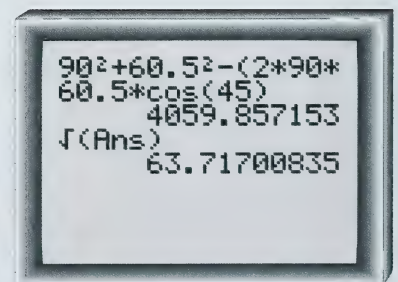
12. Draw and label a diagram to illustrate the problem.



Assess the given information and make a plan. Points P , A , and H form a triangle. Since the pitcher's mound is on the diagonal of the square, $\angle AHP$ is 45° . You know two sides and the included angle, so you can use the Cosine Law to find the length of AP (side h).



$$\begin{aligned}
 h^2 &= p^2 + a^2 - 2pa \cos H \\
 h^2 &= 90^2 + 60.5^2 - 2(90)(60.5) \cos 45^\circ \\
 h^2 &\doteq 4060 \\
 h &\doteq \sqrt{4060} \\
 &\doteq 63.72
 \end{aligned}$$



Therefore, the distance is about 63.7 feet from first base to the front of the pitcher's mound.

Extra Help

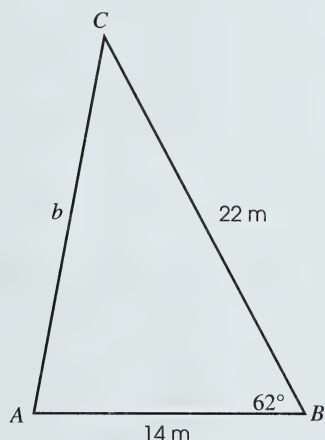
1. Your answer may or may not be reasonable. You may wish to ask someone for his or her opinion.

You may wish to find the height of easily accessed items, such as a doorway or fence. It would be easy to find the actual height of these structures.

If you wish to measure the height of structures such as your home or a public building, you may be able to check your measurements against the blueprints of the building.

2. Answers will vary. A sample answer is shown.

- a. Your diagram should look similar to the following. The measures of the angles and lengths of the triangle sides will vary.



- b. Answers will vary. The following are calculations for the sample shown in part a.

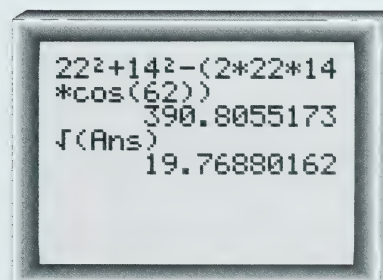
Step 1: Find the length of AC (side b). Use the Cosine Law.

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$b^2 = 22^2 + 14^2 - 2(22)(14) \cos 62^\circ$$

$$b \approx 19.8$$

The length of side b is about 20 m.



Follow-up Activities (continued)

Step 2: Find the measure of $\angle C$. Use the Sine Law. Use any two ratios where three of the four values are known.

$$\begin{aligned}\frac{\sin C}{c} &= \frac{\sin B}{b} \\ \sin C &= \left(\frac{\sin B}{b} \right) c \\ &= \left(\frac{\sin 62^\circ}{19.8} \right) 14 \\ &\doteq 0.6253\end{aligned}$$

Use the inverse function of sine to find the measure of $\angle C$.

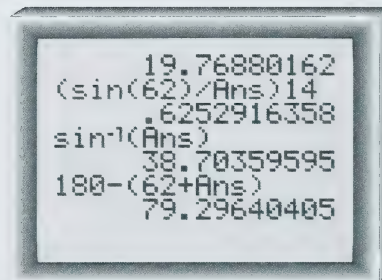
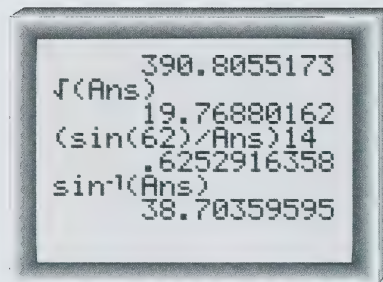
$$\angle C \doteq 38.7^\circ$$

The measure of $\angle C$ is about 39° .

Step 3: Find the measure of $\angle A$. Use the angle sum property.

$$\begin{aligned}\angle A &= 180^\circ - (\angle B + \angle C) \\ &\doteq 180^\circ - (62^\circ + 38.7^\circ) \\ &\doteq 180^\circ - 100.7^\circ \\ &\doteq 79.3^\circ\end{aligned}$$

The measure of $\angle A$ is about 79° .

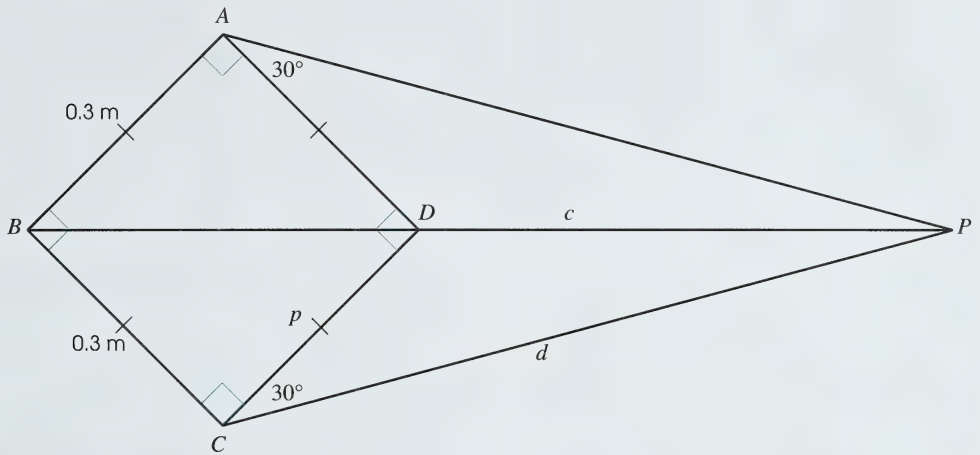


Enrichment

1. Answers will vary. How did the length of your strides and pace angles compare with the race walker's measurements? You might enjoy measuring the tracks made by other family members or, perhaps, tracks made by pets or other animals.

2. Textbook exercise 13 of Part B of “What Should I Be Able To Do?” p. 380

13. Draw and label a diagram to illustrate the problem. Assess the given information.



- a. You know the length of CD and the measure of $\angle DCP$. First, find the measures of $\angle CDP$ and $\angle DPC$.

Step 1: Angle CDB and $\angle CDP$ are supplementary angles. First, find the measure of $\angle CDB$.
Then, subtract the measure of $\angle CDB$ from 180° to find the measure of $\angle CDP$.

$$\angle ADB = \angle CDB$$

$$\angle ADC = 90^\circ, \text{ so}$$

$$\angle ADB = 45^\circ$$

$$\angle CDB = 45^\circ$$

$$\angle CDP = 180^\circ - \angle CDB \quad \leftarrow \text{supplementary angles}$$

$$= 180^\circ - 45^\circ$$

$$= 135^\circ$$

Step 2: Use the angle sum property to find the measure of $\angle DPC$.

$$\angle DPC = 180^\circ - (\angle CDP + \angle DCP)$$

$$= 180^\circ - (135^\circ + 30^\circ)$$

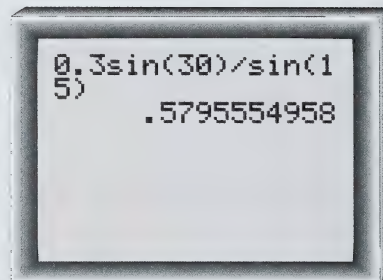
$$= 180^\circ - 165^\circ$$

$$= 15^\circ$$

Follow-up Activities (continued)

Step 3: Find the measure of PD (side c). Use the Sine Law.

$$\begin{aligned}\frac{P}{\sin P} &= \frac{c}{\sin C} \\ \frac{0.3}{\sin 15^\circ} &= \frac{PD}{\sin 30^\circ} \\ PD &= \frac{0.3 \sin 30^\circ}{\sin 15^\circ} \\ &\doteq 0.58\end{aligned}$$

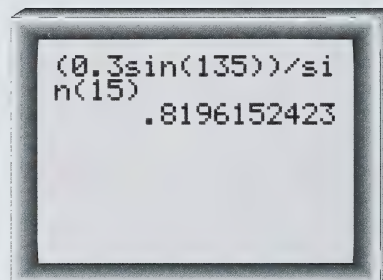


0.3sin(30)/sin(15)
.5795554958

The length of PD is about 0.6 m.

- b.** Use the Sine Law to find the length of PC (side d).

$$\begin{aligned}\frac{P}{\sin P} &= \frac{d}{\sin D} \\ \frac{0.3}{\sin 15^\circ} &= \frac{PC}{\sin 135^\circ} \\ PC &= \frac{0.3 \sin 135^\circ}{\sin 15^\circ} \\ &\doteq 0.82\end{aligned}$$



(0.3sin(135))/sin(15)
.8196152423

The length of PC is about 0.8 m.

Module Project: Land Surveying

Completing the Project

Textbook exercises 16 and 17 of Part C of “What I Should Be Able To Do?” pp. 382 and 383

- 16.** A clinometer with the angle measures on a linear scale is never accurate. The curved scale on the protractor cannot be transferred to a linear scale with equal units.

Without a plumb bob to set 0° of the clinometer to vertical, neither instrument is very precise.

The clinometer with the angles on the curve will measure angles more accurately than the linear clinometer. The curved scale shows degrees in equal units.

The instructions were not followed in constructing either clinometer. A straw for sighting has not been included and a weighted string for a plumb bob is not attached. The clinometer with the degrees on the curved line can be made to work with a straw on the top edge and a weighted string attached at point A. The other drawing does not represent a usable clinometer.

- 17.** No scale is given. You can infer a scale of $1 \text{ cm} = 2.5 \text{ m}$ from the house or pool dimensions. Some angles are clearly marked: for example, from the house toward the west fence along the fence line. However, angles from Point C are not marked. They are only mentioned in the statement of elevations. The angles marked for the trees are not accurate.

No calculations of measurements are evident.

All the given measurements are correctly taken as direct measurements rather than calculations from indirect measurements. For example, it is likely easier to measure the length of the fences, the house, and pool than to calculate the measurements.

The distances from the north and south fences to the pool could have been added. These are direct measurements. Also, the height of the house and heights of the trees are not done. These are calculated measurements.

It is not indicated whether each student included his or her height in the angle measurements taken.

Overall, the students have done a reasonable job of completing the Land Surveying project. Their drawings are neat and well labelled. The angles and distance measurements are clearly marked. Assuming the angle measurements are accurate, the students should be able to calculate accurate tree and house heights.

CREDITS

Some clip art drawings are commercially owned.

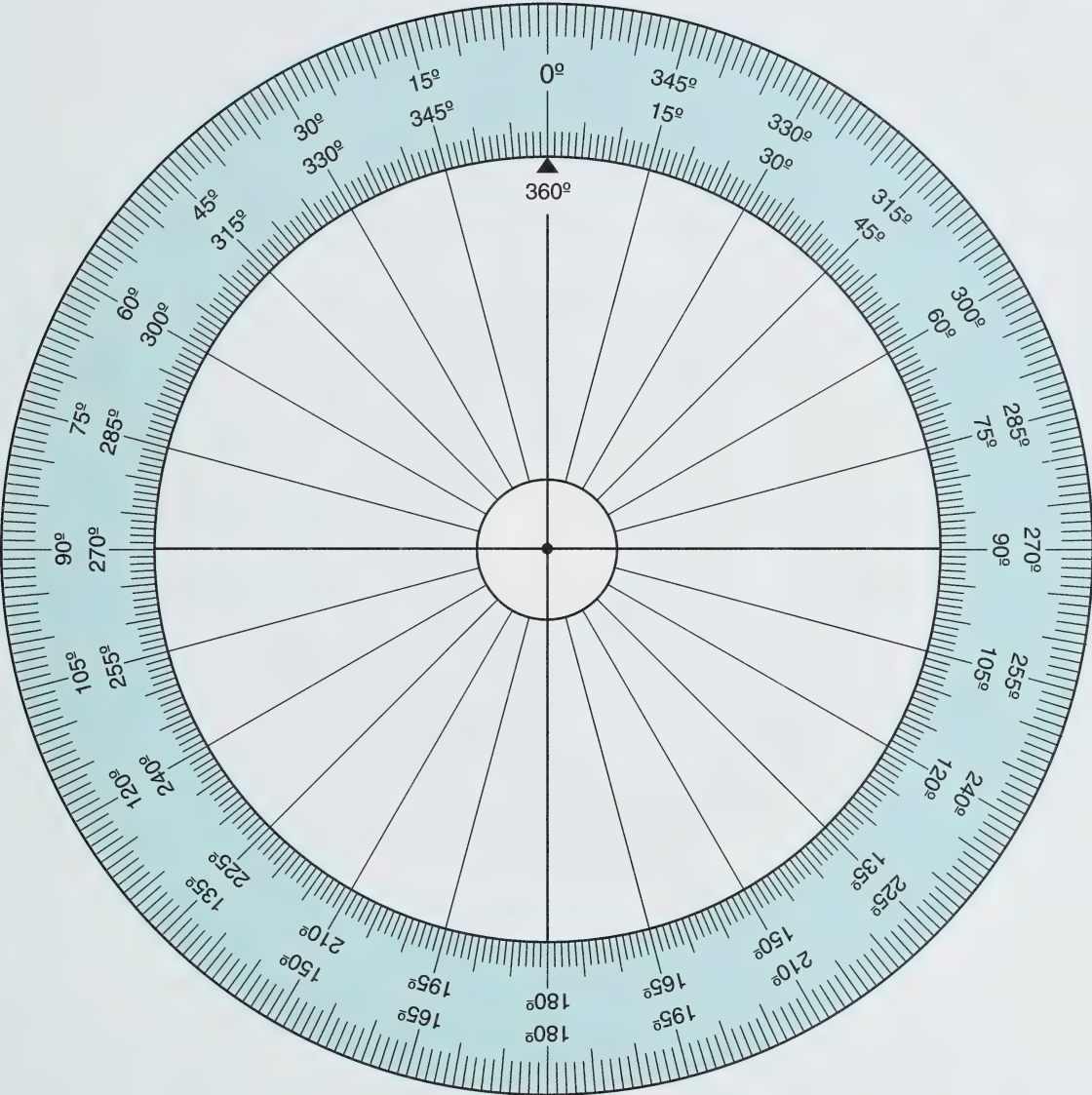
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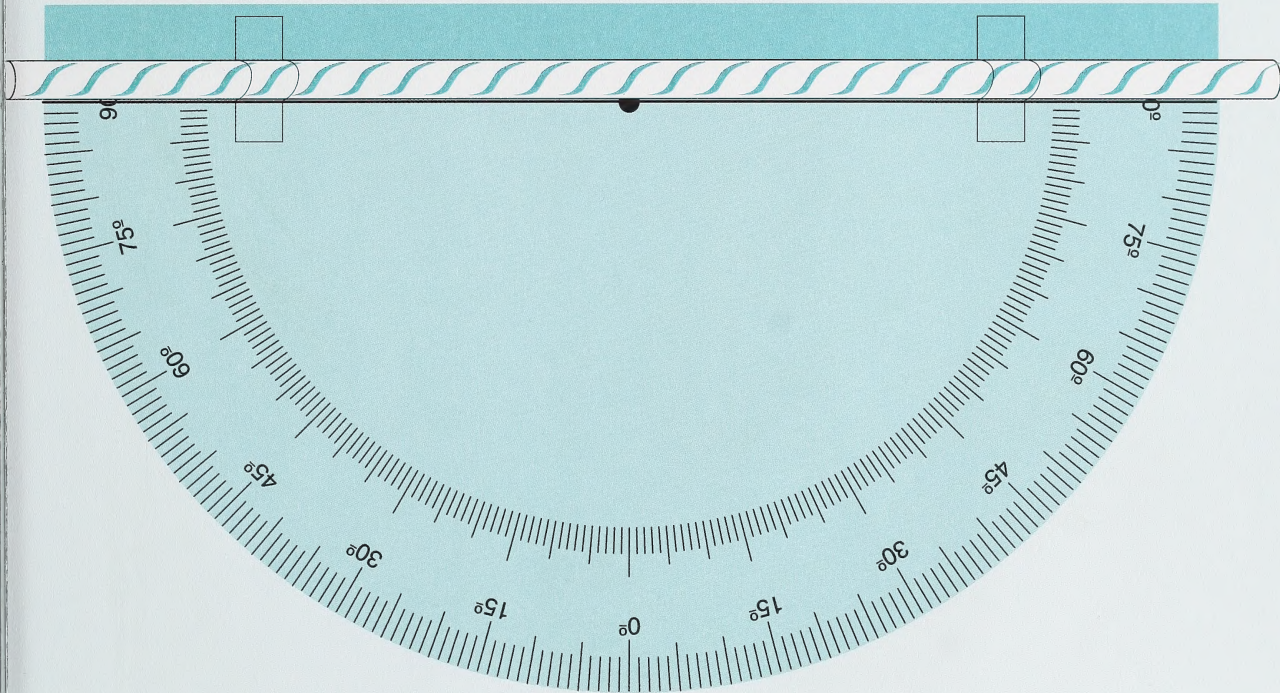
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- 15 EyeWire, Inc.
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- 35 PhotoDisc, Inc.
- 36 Image Club/StudioGear/EyeWire, Inc.
- 38 **bottom:** Image Club/StudioGear/EyeWire, Inc.
- 39 **top:** Corel Corporation
bottom: Image Club/StudioGear/EyeWire, Inc.
- 40 PhotoDisc, Inc.
- 41 **top:** Image Club/StudioGear/EyeWire, Inc.
middle: PhotoDisc, Inc.
- 43 Image Club/StudioGear/EyeWire, Inc.
- 44 **top:** Adobe Systems Incorporated
bottom: Image Club/StudioGear/EyeWire, Inc.
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- 58 PhotoDisc, Inc.
- 59 Corel Corporation

360° Protractor



180° Protractor





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